Mathematics 3770H – Complex Analysis

Trent University, Winter 2022

Assignment #5 – Around and around we go!

Due on Friday, 18 February.

(May be submitted on paper or via Blackboard.)*

As with all the assignments in this course, unless stated otherwise on the assignment, you are permitted to work together and look things up, so long as you acknowledge the sources you used and the people you worked with.

In all that follows on this assignment, let γ be the unit circle in $\mathbb{C}$ centred at the origin. The most common, but hardly the only, parametrization of this closed curve is $\gamma(\theta) = e^{i\theta}$, where $\theta \in [0, 2\pi]$.

1. For $n \in \mathbb{Z}$, show that

$$\int_\gamma z^n \, dz = \begin{cases} 0 & n \neq -1 \\ 2\pi i & n = -1 \end{cases}. \quad [6]$$

2. Given an $m \in \mathbb{Z}$, find a piecewise smooth closed path η in $\mathbb{C}$ such that

$$\int_\eta z^{-1} \, dz = 2m\pi i. \quad [4]$$

Sing me a song of the hydrogen light
Three degrees Kelvin illumine the night
Three degrees Kelvin, the infrared sky
Colors too deep for the unaided eye

Sing me a song of the hydrogen band
Whispering low since the cosmos began
Whispering low as the white light shifts red
Wavefronts of hydrogen sweeping ahead

Sing me a song of the hydrogen wall
Vector me out to that light bounding all
Vector me out in that glory to dwell
End of the universe, cosmic eggshell.

By John M. Ford, from his novel *Princes of the Air*.