

## Mathematics 3770H – Complex Analysis

TRENT UNIVERSITY, Winter 2022

### Assignment #4 – A Möbius Transformation

Due on Friday, 11 February.

(May be submitted on paper or via Blackboard.\*)

As with all the assignments in this course, unless stated otherwise on the assignment, you are permitted to work together and look things up, so long as you acknowledge the sources you used and the people you worked with.

In all that follows on this assignment, let  $a = \frac{1+i}{2}$  and let  $f : \mathbb{C} \rightarrow \mathbb{C}$  be given by  $f(z) = \frac{z-a}{1-\bar{a}z}$ .

1. Verify that  $f(z)$  is a Möbius transformation. [1]

Let  $D = \{z \in \mathbb{C} \mid |z| \leq 1\}$  be the closed unit disk in the complex plane. Note that  $D$  includes its border, *i.e.* the unit circle in the complex plane.

2. Show that  $f(z) \in D$  for every  $z \in D$  and that  $f(z)$  is 1-1 and onto when considered as a function  $D \rightarrow D$ . [6]

A *fixed point* of a complex function  $g(z)$  is a complex number  $w$  such that  $g(w) = w$ .

3. Find all the fixed points in  $\mathbb{C}$  of  $f(z)$ . [3]

### For One Who Loves An Engineer

Sing not to me of silicon chips  
The chocolate kind are sweeter  
But sweeter still would be your lips –  
Put down that voltage meter!  
A jug of wine (viscosity 3),  
Loaf of bread (shear modulus 7)  
But me you cannot quantify –  
Oh, glory be to heaven!

By Miriam Nadel

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\* All else failing, please email your solutions to the instructor at: [sbilaniuk@trentu.ca](mailto:sbilaniuk@trentu.ca)