Mathematics 3770H – Complex Analysis
TRENT UNIVERSITY, Winter 2022

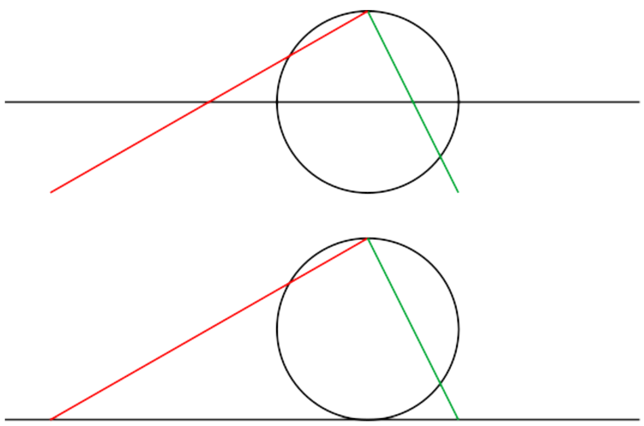
Assignment #3 – Projections
Due on Friday, 4 February.
(May be submitted on paper or via Blackboard.)*

As with all the assignments in this course, unless stated otherwise on the assignment, you are permitted to work together and look things up, so long as you acknowledge the sources you used and the people you worked with.

Please read (or at least skim :-) §3.3 in the textbook. Among other things, this section describes a way of identifying the Riemann sphere $\widehat{\mathbb{C}} = \mathbb{C} \cup \{\infty\}$, which is discussed in §3.2, with the unit sphere centred at the origin in $\mathbb{R}^3$ (*i.e.* $x^2 + y^2 + z^2 = 1$) via stereographic projection, with the xy-plane in $\mathbb{R}^3$ being identified with the complex plane via $(x, y, 0) \longleftrightarrow x + iy$. This projection works by taking the point (x, y, z) on the unit sphere to the point where the line of $\mathbb{R}^3$ joining the north pole $(1, 0, 0)$ to (x, y, z) intersects the xy-plane. The north pole itself is identified with the point at infinity, *i.e.* ∞, of the Riemann sphere.

A slightly different way of doing stereographic projection is to do the projection in the same way, but identify the complex plane with the plane $z = -1$ in $\mathbb{R}^3$ via $(x, y, -1) \longleftrightarrow x + iy$ and take the point (x, y, z) on the unit sphere to the point where the line of $\mathbb{R}^3$ joining the north pole $(1, 0, 0)$ to (x, y, z) intersects the plane $z = -1$. The north pole itself is, once again, identified with the point at infinity, *i.e.* ∞, of the Riemann sphere.

Cross-sectional views of how these projections work are pictured in the diagram below.



Let's call the projection that takes points on the unit sphere to the Riemann sphere in the first version of stereographic projection given above ϕ and the projection that takes points on the unit sphere to the Riemann sphere via the second version of stereographic projection given above by ψ.

1. What exactly is the mapping of the Riemann sphere to itself given by $\psi \circ \phi^{-1}$? Is it 1-1 and onto? Is it differentiable? Is it a Möbius transformation? *[10]*

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* All else failing, please email your solutions to the instructor at: `sbilaniuk@trentu.ca`