A little differentiability
Due on Friday, 28 January.
(May be submitted on paper or via Blackboard.*)

As with all the assignments in this course, unless stated otherwise on the assignment, you are permitted to work together and look things up, so long as you acknowledge the sources you used and the people you worked with.

1. Suppose $f(z) = u(x, y) + iv(x, y)$, where $z = x + iy$ for real numbers x and y, and u and v are real-valued functions of two real variables which are differentiable at all $(x, y) \in \mathbb{R}^2$. Either show that it follows that $f(z)$ is differentiable as a function $\mathbb{C} \to \mathbb{C}$ or find a counterexample. *[5]*

2. Suppose $f : \mathbb{C} \to \mathbb{C}$ is differentiable on all of $\mathbb{C}$ and $f(z) \in \mathbb{R}$ for every $z \in \mathbb{C}$, *i.e.* $f(z)$ happens to output only real numbers. Show that $f(z)$ is a constant function. *[5]*

Love and Tensor Algebra

Come, let us hasten to a higher plane,
Where dyads tread the fairy fields of Venn,
Their indices bedecked from one to n,
Commingled in an endless Markov chain!

Come, every frustrum longs to be a cone,
And every vector dreams of matrices.
Hark to the gentle gradient of the breeze:
It whispers of a more ergodic zone.

In Riemann, Hilbert or in Banach space
Let superscripts and subscripts go their ways.
Our asymptotes no longer out of phase,
We shall encounter, counting, face to face.

I'll grant thee random access to my heart,
Thou'lt tell me all the constants of thy love;
And so we two shall all love's lemmas prove,
And in our bound partition never part.

For what did Cauchy know, or Christoffel,
Or Fourier, or any Boole or Euler,
Wielding their compasses, their pens and rulers,
Of thy supernal sinusoidal spell?

Cancel me not for what shall then remain?
Abscissas, some mantissas, modules, modes,
A root or two, a torus and a node:
The inverse of my verse, a null domain.

Ellipse of bliss, converge, O lips divine!
The product of our scalars is defined!
Cyberiad draws nigh, and the skew mind
Cuts capers like a happy haversine.

I see the eigenvalue in thine eye,
I hear the tender tensor in thy sigh.
Bernoulli would have been content to die,
Had he but known such $a^2 cos(2\phi)$!

By Stanislaw Lem, from his story collection *The Cyberiad*, translated from the Polish by Michael Kandel.

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*   All else failing, please email your solutions to the instructor at: `sbilaniuk@trentu.ca`