Mathematics 3770H – Complex Analysis
Trent University, Winter 2022

Assignment #11
Due on Friday, 8 April.
(May be submitted on paper or via Blackboard.)*

As with all the assignments in this course, unless stated otherwise on the assignment, you are permitted to work together and look things up, so long as you acknowledge the sources you used and the people you worked with.

Recall from Assignment #10 that if $r \in \mathbb{C}$ and $k \geq 1$ is an integer, then the *binomial of* r *and* k is

$$\binom{r}{k} = \frac{r(r-1)(r-2)\cdots(r-k+1)}{k!} .$$

Thus $\binom{r}{1} = r$, $\binom{r}{2} = \frac{r(r-1)}{2}$, $\binom{r}{3} = \frac{r(r-1)(r-2)}{6}$, and so on. To make various formulas work nicely, we let $\binom{r}{0} = 1$. Note that when r is a positive integer, this coincides with the usual definition of binomial coefficients.

1. Suppose a, $r \in \mathbb{C}$ with $a \neq 0$. Using Newton's Binomial Theorem, we can expand $(z+a)^r$ as a more or less Laurent series as follows:

$$\begin{aligned}(z+a)^r &= z^r + raz^{r-1}a + \frac{r(r-1)}{2}a^2z^{r-2} + \frac{r(r-1)(r-2)}{6}a^3z^{r-3} + \cdots \\ &= \sum_{n=0}^{\infty} \binom{r}{n} a^n z^{r-n} = z^r \sum_{n=0}^{\infty} \binom{r}{n} \left(\frac{a}{z}\right)^n\end{aligned}$$

Find the region within which this series converges absolutely. *[10]*

* All else failing, please email your solutions to the instructor at: `sbilaniuk@trentu.ca`