Mathematics 3770H – Complex Analysis
Trent University, Winter 2022

Assignment #10
Due on Friday, 25 March.
(May be submitted on paper or via Blackboard.)*

As with all the assignments in this course, unless stated otherwise on the assignment, you are permitted to work together and look things up, so long as you acknowledge the sources you used and the people you worked with.

Suppose $r \in \mathbb{C}$ and $k \geq 1$ is an integer. Then the *binomial of* r *and* k is

$$\binom{r}{k} = \frac{r(r-1)(r-2)\cdots(r-k+1)}{k!}$$

and so we have $\binom{r}{1} = r$, $\binom{r}{2} = \frac{r(r-1)}{2}$, $\binom{r}{3} = \frac{r(r-1)(r-2)}{6}$, and so on. Somewhat arbitrarily to make various formulas work nicely, we let $\binom{r}{0} = 1$. Note that when r is a positive integer, this coincides with the usual definition of binomial coefficients. It's a bit harder to justify reading $\binom{r}{k}$ as "r choose 0" when r is not a positive integer, though.

1. Suppose a, $r \in \mathbb{C}$ with $a \neq 0$. Using Newton's Binomial Theorem, we can expand $(a+z)^r$ as a power series as follows:

$$(a+z)^r = a^r + ra^{r-1}z + \frac{r(r-1)}{2}a^{r-2}z^2 + \frac{r(r-1)(r-2)}{6}a^{r-3}z^3 + \cdots = \sum_{n=0}^{\infty} \binom{r}{n} a^{r-n} z^n$$

Determine the radius of convergence of this power series. *[7]*

2. Give at least two uses for expansions of this form of expressions like $(a+z)^r$. *[3]*

* All else failing, please email your solutions to the instructor at: `sbilaniuk@trentu.ca`