

Mathematics 3770H – Complex Analysis

TRENT UNIVERSITY, Winter 2022

A little algebra with complex numbers

Due on Friday, 21 January.

(May be submitted on paper or via Blackboard.*)

This assignment is mainly meant as a warm-up exercise. As with all the assignments in this course, unless stated otherwise on the assignment, you are permitted to work together and look things up, so long as you acknowledge the sources you used and the people you worked with.

1. Prove *Euler's Formula*, that $e^{i\theta} = \cos(\theta) + i\sin(\theta)$, by using the Taylor series at 0 of e^x , $\cos(x)$, and $\sin(x)$ and some algebra. [3]
2. Prove *De Moivre's Theorem*, that $[\cos(\theta) + i\sin(\theta)]^n = \cos(n\theta) + i\sin(n\theta)$ for all integers $n \geq 0$, with the help of Euler's formula. [1]
3. Prove *De Moivre's Theorem*, that $[\cos(\theta) + i\sin(\theta)]^n = \cos(n\theta) + i\sin(n\theta)$ for all integers $n \geq 0$, *without* using Euler's formula. [3]
4. Suppose you are given the complex numbers se^α and te^β . As best you can, find expressions for r and θ in terms of s , t , α , and β such that $re^\theta = se^\alpha + te^\beta$. [3]

Ode to a ϕ ?

A thing as lovely as a ϕ

I think that I shall never $\left\{ \begin{array}{l} \text{see} \\ \text{spy} \end{array} \right\}$. [Choose one.]

If you want to make ϕ rhyme with “see”,

But you're really determined to be

Consistent, admit

Though it doesn't quite fit,

That a disk measures r^2 times “pea”.

Each of ξ , π , ϕ , χ , ψ will claim

An identical rhyme for its name.

Before you get hot,

Check Liddell and Scott[†];

Give them, not the author, the blame.

By Barry W. Brunson, *American Mathematical Monthly* **92** (1985).

* All else failing, please email your solutions to the instructor at: sbilaniuk@trentu.ca

[†] *A Greek-English Lexicon*, compiled by H.G. Liddell & R. Scott, Oxford at the Clarendon Press, 1968 (1st edition 1843); widely recognized as authoritative. In the United States, most mathematicians seem to pronounce ϕ as “fee” rather than “fie”. Liddell was the father of Alice Liddell, for whom Lewis Carroll wrote *Alice's Adventures in Wonderland* and *Through the Looking-Glass*.