

Mathematics 3260H – Geometry II: Projective and non-Euclidean geometry
TRENT UNIVERSITY, Winter 2015

Quizzes

Quiz #1. Monday, 12 January, 2015. [10 minutes]

Euclid's five postulates are:

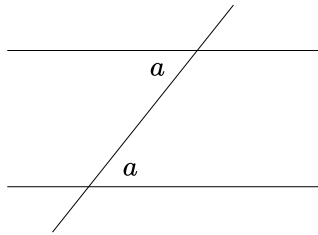
- I. One can draw a straight line from any point to any point.
- II. One can produce a finite straight line continuously in a straight line.
- III. One can describe a circle with any centre and distance.
- IV. All right angles are equal to one another.
- V. If a straight line falling on two straight lines makes the sum of the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles whose sum is less than two right angles.

(Those who took Euclidean geometry probably remember that one needs to add some additional assumptions beyond those Euclid gives to make some of his proofs work, but we won't bother with those here.)

1. Which of Euclid's axioms does the real line (with the points on it, of course) satisfy? Explain, briefly and informally, why you think so. [5]

Quiz #2. Monday, 19 January, 2015. [10 minutes]

You may assume that the *Z-Theorem*, that if a line crosses two parallel lines, then alternate interior angles are equal, is true.

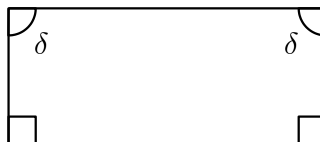


1. Show that if the Playfair's axiom (the most common version of the parallel axiom) is true, then the sum of the interior angles of a triangle is equal to two right angles. [5]

Quiz #3. Monday, 26 January, 2015. [10 minutes]

Recall that a hyperbolic triangle with interior angles α , β , and γ (in radians) has (for a suitable choice of units) area $\pi - \alpha - \beta - \gamma$.

1. Suppose we are given a Saccheri quadrilateral in the hyperbolic plane with upper interior angles both equal to δ .



Determine the area of this Saccheri quadrilateral. [5]

Quiz #4. ~~Monday, 2 Tuesday, 3~~ Thursday, 5 February, 2015. [10 minutes]

Recall that the area of an elliptic triangle with interior angles α , β , and γ (in radians) is (for a suitable choice of units) its excess, namely $\alpha + \beta + \gamma - \pi$.

1. What is the maximum area of an elliptic triangle? [4]
2. What is the area of the entire elliptic plane? [1]

Hint for both: Use the hemispherical model of the elliptic plane.

Quiz #5. Monday, 9 February, 2015. [10 minutes]

In the problems below, you may assume that units of length have been chosen in the elliptic plane to make the area of a triangle be $\alpha + \beta + \gamma - \pi$ if its internal angles α , β , and γ are measured in radians.

1. What is the maximum length of a line segment in the elliptic plane? [3]
2. What is the maximum distance between two points in the elliptic plane? [2]

Quiz #6. Monday, 23 February, 2015. [10 minutes]

Recall from the nebulous past before reading week that an *affine plane* is a structure consisting of *points* and *lines*, together with a relation of *incidence* between points and lines satisfying the following axioms:

- AI.** Any two distinct points are incident with a unique line.
 - AII.** Given a point P and a line ℓ not incident with P , there is a unique line m incident with P which has no point in common with ℓ .
 - AIII.** There exist three points which are not incident with the same line.
1. Show that every line of an affine plane must be incident with at least two points. [5]

Quiz #7. Monday, 2 March, 2015. [10 minutes]

Recall that a *projective plane* is a structure consisting of points and lines, together with a relation of incidence between points and lines satisfying the following axioms:

- I.** Any two distinct points are incident with a unique line.
 - II.** Any two distinct lines are incident with a unique point.
 - III.** There exist four points such that no three are incident with the same line.
1. Suppose that a projective plane actually has five points such that no three are incident with the same line. Show that it must have five lines such that no three are incident with the same point. [5]

Quiz #8. Monday, 9 March, 2015. [10 minutes]

1. Consider the triangle in the real projective plane whose vertices have projective coordinates $(1, 1, 1)$, $(1, 2, 1)$, and $(2, 1, 1)$. Find the projective coordinates of the lines forming the sides of the triangle. [5]

Quiz #9. Monday, 17 March, 2015. [10 minutes]

1. Suppose γ is a collineation of a projective plane such that $\gamma^2 = \gamma \circ \gamma = i$ is the identity collineation. Show that there is a point P such that $P^\gamma = P$. [5]

Quiz #10. *Monday, 24 March, 2015. [10 minutes]*

1. Consider the points of the real projective plane with extended affine coordinates $A = (0, 0)$, $B = (1, 0)$, $C = (0, 1)$, $P = (1, 1)$, $Q = (1, 2)$, and $R = (2, 1)$; then $\triangle ABC$ and $\triangle PQR$ are in perspective from the point (1) . By Desargues' Theorem, they must therefore be in perspective from a line. Which line is it? [5]