Mathematics 3260H – Geometry II: Projective and non-Euclidean geometry
Trent University, Winter 2015
Assignment #1
With apologies to Prof. Tolkien . . .
Due on Thursday, 15 January, 2015.

If the Númenoreans had been mathematicians, perhaps the rhyme of lore* Gandalf quotes to Pippin during the ride from Rohan to Gondor in the *The Lord of the Rings* would have been something like:

> *Tall ships and tall kings*
> *Three times three,*
> *What brought they from the foundered land*
> *Over the flowing sea?*
> Seven points and seven lines
> In one geometry:
> Every point met three lines,
> Every line met points three,
> Every pair of points connected,
> Every line pair intersected.
> Fano found it, Fano named it.

1. Draw a picture of this alternate universe Númenorean geometry, commonly known as the *Fano plane* or *Fano configuration*. *[3]*

2. Can the Fano configuration be drawn in the Euclidean plane with all of its "lines" being straight line segments of the Euclidean plane? Give an example or prove it can't be done. *[4]*

3. Is there a *quadrilateral* - four lines, no three of which pass through the same point - in the Fano plane? Give an example or show that there isn't one. *[1]*

4. Consider the following adaptation of a more famous verse† from the *Lord of the Rings*.

   > One line to meet them all, at infinity to find them,
   > One line to join them all and at a point to bind them,
   > In projective planes where parallels die.

   What mathematical object(s) or operation(s) does this verse allude to? *[2]*

Bonus: What does the most popular way to draw the Fano configuration have to do with the world of *Harry Potter*? *[0.5]*

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* "Tall ships and tall kings/ Three times three,/ What brought they from the foundered land/ Over the flowing sea?/ Seven stars and seven stones/ And one white tree."

† "One Ring to rule them all, One Ring to find them, / One Ring to bring them all and in the darkness bind them, / In the Land of Mordor where the Shadows lie."