Mathematics 326H – Geometry II: Projective and non-Euclidean geometries
TRENT UNIVERSITY, Winter 2007

Problem Set #5
Due on Thursday, 22 March, 2007.

Linear transformations and the real projective plane

Recall that a 3×3 matrix

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

with real entries can be used to define a linear transformation of $\mathbb{R}^3$ by sending $\mathbf{x}$ to $\mathbf{Ax}$.

1. What condition(s) must the matrix $\mathbf{A}$ satisfy for the corresponding linear transformation, when applied to the projective coordinates of the points of the real projective plane, to be a function that maps points of the real projective plane to the points of the real projective plane? *[3]*

2. What condition(s) must the matrix $\mathbf{A}$ satisfy for the corresponding linear transformation to be a 1-1 onto function on the points of the real projective plane? *[1]*

3. When do two matrices $\mathbf{A}$ and $\mathbf{B}$ give the same function on the points of the real projective plane? *[2]*

4. Suppose the matrix $\mathbf{A}$ gives a 1-1 onto function on the points of the real projective plane. Work out how the projective coordinates of lines are transformed by this function and show that it preserves incidence. *[4]*

Bonus. Suppose a 1-1 onto function maps the points and lines of the real projectove plane to the points and lines, respectively, of the real projective plane plane, and also preserves incidence. (That is, it is a *collineation* of the real projective plane.) Is there necessarily a matrix $\mathbf{A}$ which gives this function? *[5]*