Mathematics 326H – Geometry II: Projective and non-Euclidean geometries
TRENT UNIVERSITY, Winter 2007

Problem Set #3
Due on Tuesday, 27 February, 2007.

1. Show that for any pair of parallel lines in the Poincaré model there is an unique line perpendicular to both. *[5]*

The MAX-imum pain – er, plain – er, plane

Let MAX be the geometric structure defined with reference to the surface in three-dimensional Cartesian space given by $z^2 = x^2 + y^2 + 1$ and $z > 0$ (*i.e.* $z = \sqrt{x^2 + y^2 + 1}$) as follows.

- Points of MAX are the points of the surface.
- Lines of MAX are the curves in which the surface intersects (some of) the Cartesian planes through the origin.
- The distance in MAX between two points $\mathbf{a} = (a_1, a_2, a_3)$ and $\mathbf{b} = (b_1, b_2, b_3)$ of MAX is given by the formula:

$$d(\mathbf{a}, \mathbf{b}) = \operatorname{arccosh}(a_3 b_3 - a_1 b_1 - a_2 b_2)$$

  (You may assume that distances between points of MAX are positive and have the usual basic properties distance functions ought to.)
- The angle φ in MAX between the lines of MAX obtained by intersecting the surface with the Cartesian planes $ax + by + cz = 0$ and $px + qy + rz = 0$ is given by the formula:

$$\varphi = \arccos\left(\frac{cr - ap - bq}{\sqrt{c^2 - a^2 - b^2} \cdot \sqrt{r^2 - p^2 - q^2}}\right)$$

  (You may assume that this formula makes sense whenever two lines of MAX intersect.)

2. Determine which among Euclid's Postulates **I**–**IV** and the "multiple parallels" counterpart of Playfair's Axiom hold in MAX. *[10]*