**Mathematics 326H – Geometry II: Projective and non-Euclidean geometries**
Trent University, Winter 2007

**Problem Set #1**
*Due on Thursday, 26 January, 2007.*

**1.** $\mathbb{Z}_2$ is the number system with 0 and 1 as its only elements, and with addition and multiplication done modulo 2. ($\mathbb{Z}_2$'s arithmetic is normal except that $1 + 1 = 0 \dots$ ) $\mathbb{Z}_2^2$ is the counterpart of the Cartesian plane $\mathbb{R}^2$, constructed in the same way, apart from using $\mathbb{Z}_2$ instead of $\mathbb{R}$.

**a.** How many points and lines, respectively, are there in $\mathbb{Z}_2^2$? *[1]*

**b.** Which of Euclid's postulates make sense and are true, respectively, in $\mathbb{Z}_2^2$? *[5]*

**2.** The axioms for a projective plane are:

**I.** Any two distinct points are incident with exactly one common line.

**II.** Any two distinct lines are incident with exactly one common point.

**III.** There are four disctinct points such that no three are incident with a common line.

Give examples of structures to demonstrate that these three axioms are mutually consistent and also independent of one another. *[8]*

**3.** There are three more-or-less common definitions of what "parallel" means:

*i.* Two lines are parallel if they never meet.

*ii.* Two lines are parallel if the distance between them is constant.

*iii.* Two lines are parallel if corresponding angles are the same when the two lines are crossed by a third line.

Give (informal!) arguments to show that these definitions are equivalent in Euclidean geometry. *[6]*