Mathematics 3260H – Geometry II: Projective and Non-Euclidean Geometry

TRENT UNIVERSITY, Fall 2021

Assignment #8

Central Collineations Meet the Real Projective Plane

Due on Friday, 12 November.

*May be submitted on paper or via Blackboard.**

Recall from class that we have several equivalent ways of presenting the real projective plane:

- Adding a line at infinity to the Euclidean plane, with a point on this line where each class of parallel lines of the Euclidean plane meet.
- The coordinate version of the above, *i.e.* extended affine coordinates starting from the usual Cartesian coordinates.
- The geometry of one- and two-dimensional subspaces of $\mathbb{R}^3$, where the "points" are lines throught he origin and the "lines" are planes through the origin, with incidence being given by inclusion.
- The coordinate version of the above, *i.e.* projective or homogeneous coorcinates, where the coordinates of points are the direction vectors of the corresponding lines and the coordinates of lines are normal vectors of the corresponding planes, with a point and line being incident exactly when the dot product of the corresponding vectors is zero.

In answering the following questions, you may use whichever presentation(s) of the real projective plane you like, as long as you get the job done.

1. Suppose (P, ℓ) and (Q, m) are both non-incident point-line pairs in the real projective plane. Show that there is a collineation α, not necessarily central, such that $P^\alpha = Q$ and $\ell^\alpha = m$. *[5]*

Recall from class that a projective plane is (P, ℓ)-transitive for a point P and line ℓ if whenever X and Y are points collinear with P, but not equal to P or on ℓ, there is a (P, ℓ)-central collineation α such that $X^\alpha = Y$.

2. Show that the real projective plane is (P, ℓ)-transitive for every non-incident point-line pair (P, ℓ). *[5]*

* All else failing, please email your solutions to the instructor at: `sbilaniuk@trentu.ca`