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Mathematics 3260H – Geometry II: Projective and Non-Euclidean Geometry
Trent University, Fall 2021

Assignment #5
Not an affine plane ...
Due on Friday, 15 October.
*May be submitted on paper or via Blackboard.**

This assignment is a follow-up of sorts to Assignment #4.

$\mathbb{Z}_4$ is the algebraic structure (it's a ring, if you want to be technical) of the integers mod 4. That is, $\mathbb{Z}_4$ has four elements, 0, 1, 2, and 3, and the operations of $+$ and $\cdot$ "roll over" when they reach 4 like a clock with four hours. (Thus $4 = 0 \mod 4$, $5 = 1 \mod 4$, and so on.) To be completely explicit, the operations of $+$ and $\cdot$ in $\mathbb{Z}_4$ are given by the following tables:

+	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

·	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	0	2
3	0	3	2	1

We can use $\mathbb{Z}_4$ as the basis for a two-dimensional system of Cartesian-style coordinates, denoting the resulting geometry by $\mathbb{Z}_4^2$. Points are given by their coordinates and lines are given by equations of the form $x = a$ and $y = mx + b$. (Of course, we are restricted to using the elements and operations of $\mathbb{Z}_4$.) This gives a geometry that isn't an affine plane, as you will show.

1. Draw the "plane" $\mathbb{Z}_4^2$. *[5]*

2. Explain why $\mathbb{Z}_4^2$ is not an affine plane. List as many reasons why it isn't as you can find. *[5]*

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* All else failing, please email your solutions to the instructor at: `sbilaniuk@trentu.ca`