.

Mathematics 3260H – Geometry II: Projective and Non-Euclidean Geometry

TRENT UNIVERSITY, Fall 2021

Assignment #2

The Moulton Plane

Due on Friday, 1 October.

May be submitted on paper or via Blackboard.[*]

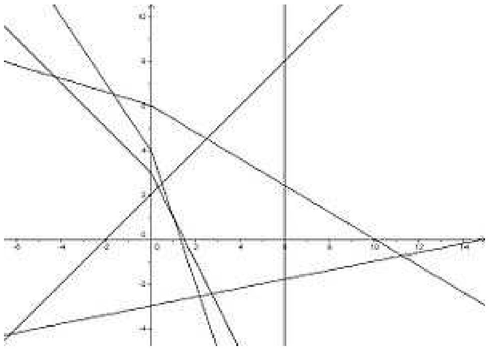
Recall from class that an affine plane is a geometry consisting of a set of points and lines satisfying the following axioms:

AI. Any two distinct points are connected by an unique line.

AII. Given a line ℓ and a point P not on ℓ, there is an unique line m through P that has no points in common with ℓ.

AIII. There exist three points that are not all on the same line.

The *Moulton plane*[†] is the affine plane obtained from the Cartesian plane by replacing straight lines with negative slope by lines which bend to double the slope as they cross the y-axis from left to right.



More formally:

- The points of the Moulton plane are the points of the Cartesian plane $\mathbb{R}^2$.
- The lines of the Moulton plane are:
  - The vertical lines of the Cartesian plane, *i.e.* $x = c$ for each $c \in \mathbb{R}$.
  - The lines of non-negative slope of the Cartesian plane, *i.e.* $y = mx + b$ for $m, b \in \mathbb{R}$ with $m \geq 0$.
  - The bent lines given by $y = \begin{cases} mx + b & x \leq 0 \\ 2mx + b & x \geq 0 \end{cases}$ for $m, b \in \mathbb{R}$ with $m \leq 0$.
- A point is on a line of the Moulton plane exactly when its Cartesian coordinates satisfy the equation of the line.

1. Verify that the Moulton plane is indeed an affine plane. *[10]*

---

[*] All else failing, please email your solutions to the instructor at: `sbilaniuk@trentu.ca`

[†] Named after the astronomer (!) who devised this example, Forest Ray Moulton (1872–1952). In mathematics, Moulton is also known for the Adams-Moulton methods for computing numerical solutions to differential equations.