

**Mathematics 3260H – Geometry II: Projective and Non-Euclidean Geometry**

TRENT UNIVERSITY, Fall 2021

**Assignment #2**

**Making an affine plane out of a projective plane**

*Due on Friday, 24 September.*

*May be submitted on paper or via Blackboard.\**

Recall from class that an affine plane is a geometry (technically, a *configuration*) consisting of a set of points and lines, and a relation of incidence between them, satisfying the following axioms:

- AI.** Any two distinct points are connected by a unique line.
- AII.** Given a line  $\ell$  and a point  $P$  not on  $\ell$ , there is a unique line  $m$  through  $P$  that has no points in common with  $\ell$ .
- AIII.** There exist three points that are not all on the same line.

Recall also that a projective plane is a geometry consisting of a set of points and lines, and a relation of incidence between them, satisfying the following axioms:

- I.** Any two distinct points are connected by a unique line.
- II.** Any two distinct lines meet at a unique point.
- III.** There exist four points such that no three are on the same line.

It turns out that these definitions are very closely related, as one can convert one type of plane into the other by a fairly simple procedure.

1. Suppose you are given a projective plane and proceed to remove a single line and all the points on this line from the given plane. Show that what is left over is an affine plane. [10]

NOTE. This is the complementary result to the construction of a projective plane by adding a “line at infinity” to an affine plane.

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\* All else failing, please email your solutions to the instructor at: [sbilaniuk@trentu.ca](mailto:sbilaniuk@trentu.ca)