Assignment #11
Saccheri Quadrilaterals
Due on Friday, 3 December.

A *Saccheri quadrilateral* is a quadrilateral $ABCD$ in which sides AB and CD are perpendicular to the base BC, with A and D on the same side of BC, and with $AB = CD$ (*i.e.* AB and CD have the same length).

1. Use Euclid's Postulates I–IV (augmented by Postulates A and S, if you need them) to show that $\angle BAD = \angle CDA$. *[4]*

2. Suppose that in our Saccheri quadrilateral we have $AB = BC = CD = 1$. Show that:
   a. $AD < 1$ if the quadrilateral is drawn in the elliptic plane. *[2]*
   b. $AD = 1$ if the quadrilateral is drawn in the Euclidean plane. *[2]*
   c. $AD > 1$ if the quadrilateral is drawn in the hyperbolic plane. *[2]*

NOTE: Giovanni Saccheri (1667-1733) was a Jesuit priest who did work in theology, philosophy, and mathematics. He is best remembered nowadays for his work *Euclides ab omni naevo vindicatus* ("Euclid cleared of every flaw"), published shortly before he died. In this book he attempted to show that Euclid's Postulate V followed from Postulates I–IV by obtaining a contradiction from assuming Postulates I–IV and denying Postulate V. He succeeded in showing that denying Postulate V by assuming that there are no parallel lines led to a contradiction, but his argument that denying the Postulate by assuming that there were many lines through a given point parallel to a given line was flawed. However, he proved many theorems of what is now known as hyperbolic geometry along the way.