

Mathematics 3260H – Geometry II: Projective and Non-Euclidean Geometry

TRENT UNIVERSITY, Fall 2021

Assignment #1

An Odd Geometry

Due on Friday, 17 September.

*May be submitted on paper or via Blackboard.**

A geometry, call it \mathbf{G} , is defined as follows.

1. The points of \mathbf{G} are the points of the sphere $x^2 + y^2 + (z - 1)^2 = 1$, except that the origin, *i.e.* the point $(0, 0, 0)$, is *not* a point of \mathbf{G} .
2. The lines of \mathbf{G} are the curves on the sphere $x^2 + y^2 + (z - 1)^2 = 1$ (with the origin deleted) that are the intersections of the sphere with planes through the origin (other than the plane $z = 0$, which does not pass through any other point of the sphere).
3. A point of \mathbf{G} is on a line of \mathbf{G} has the usual meaning, *i.e.* the point of the sphere is on the curve on the sphere.
4. Lines of \mathbf{G} intersect if they have a common point.
5. Angles between lines of \mathbf{G} at a point of intersection are the angles between the planes defining these lines.

1. Determine which of Euclid's five Postulates are true in \mathbf{G} . If a postulate is true in \mathbf{G} , give an (informal) explanation of why this is the case; if a postulate is false in \mathbf{G} , *i.e.* it fails at least once, give a specific example of such a failure. [10]

HINT. Think back to MATH 2200H and problem-solving. What would Polya do?

* All else failing, please email your solutions to the instructor at: sbilaniuk@trentu.ca