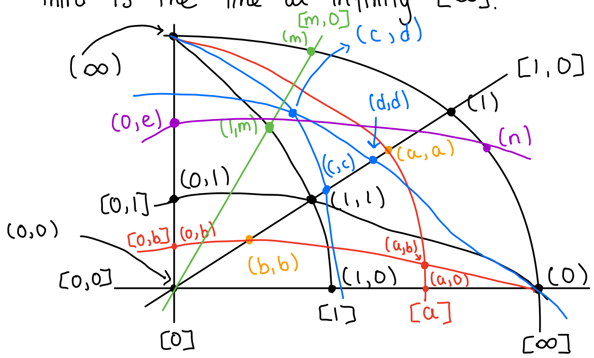
Coordinating ... from the geometry alone (for projective planes)

- Start with a projective plane. We need a collection of symbols to populate the coordinates.
- Three special ones: ∞, 0, 1.
- We also need as many more as there points on a line (when counted with the three special ones)
- Given the plane choose three lines that are not concurrent in a single point.
- One will be the line $y=0$ $[0,0]$, one will be the line $x=0$ $[0]$, and the third is the line at infinity $[\infty]$.



- We assign coordinates to 3 points:

  $(0,0) = [0] \cap [0,0]$

  $(0) = [\infty] \cap [0,0]$

  $(\infty) = [\infty] \cap [0]$
- Pick a point on $[\infty]$ other than (0) or (∞) and call it (1).
- The line joining $(0,0)$ to (1) is given coordinates $[1,0]$.
- Pick a point on $[1,0]$ other than (1) and $(0,0)$ and call it $(1,1)$.
- The line joining (∞) to $(1,1)$ must be $[1]$ and meets $[0,0]$ at $(1,0)$.
- The line joining (0) to $(1,1)$ has coordinates $[0,1]$ and it meets $[0]$ at $(0,1)$
- To every point on $[1,0]$ for which we don't already have coordinates assign coordinates (a,a) for one of the symbols not used yet. [Use a different symbol for each different point].
- The line joining (∞) to (a,a) gets coordinates $[a]$ and the line joining (0) to (b,b) get coordinates $[0,b]$
- To find the coordinates of any other point not on $[\infty]$
- Connect it to (0), this gives you a line passing through (d,d) on $[1,0]$, and connect it to (∞), this gives a line passing through (c,c) on $[1,0]$, giving the point we started with coordinates (c,d)
- For the points not yet taken care of on $[\infty]$, connect the point to $(0,0)$ and look at the intersection of this line with $[1]$. If the intersection has coordinates $(1,m)$ the line is $[m,0]$ and meets our point on $[\infty]$, giving it the coordinate (m).
- The line passing through (n) and $(0,e)$ gets coordinates $[n,e]$ and then we're done ... except for defining the algebraic operations.