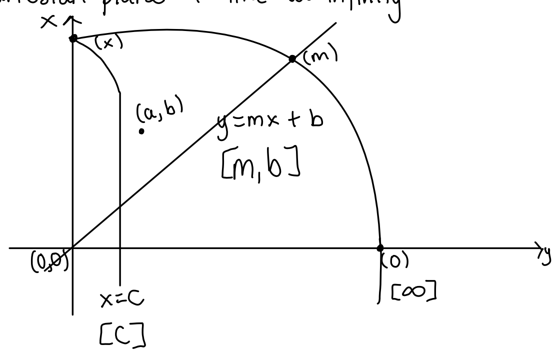
What do we need to do to tell if two projective planes are the same as incident structures?

Starring the real projective plane in two incarnation!

Two presentations of the real projective plane:

1. Cartesian plane & "line at infinity"



2. using planes through the origin as "lines" and lines through the origin as "points"

↓ normal vectors $[a,b,c]$ (not all 0) ↓ direction vectors (x,y,z) (not all 0)

Incidence is given by $[a,b,c](x,y,z) = \lambda(ax+by+cz) = 0$
$(\lambda \neq 0)$

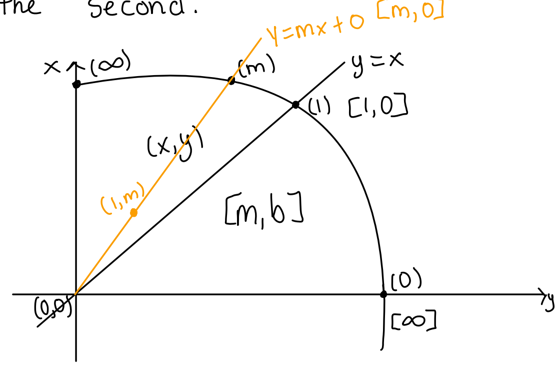
One odd feature: If we multiply the coordinates of a point or line by non-zero constant it's the same point or line.

We claim these are the same structure as far as incidence is concerned.

What do we need to do to check this?

↳ We need a 1-1 onto correspondence between the points in each presentation, and a 1-1 onto correspondence between the lines in each one, and it should "preserve incidence".

We'll start with the first presentation and establish a correspondence with the second.



$(x,y) \longrightarrow (x, y, 1)$

$(m) \longrightarrow (1, m, 0)$

$(\infty) \longrightarrow (0, 1, 0)$

$[m,b] \longrightarrow [-m, 1, -b]$

$[c] \longrightarrow [1, 0, -c]$

$[\infty] \longrightarrow [0, 0, 1]$

$(0,0) \rightarrow (0,0,1)$

$(1,m) \rightarrow (1,m,1)$

$[m,0] \rightarrow [-m,1,0]$ → dot product

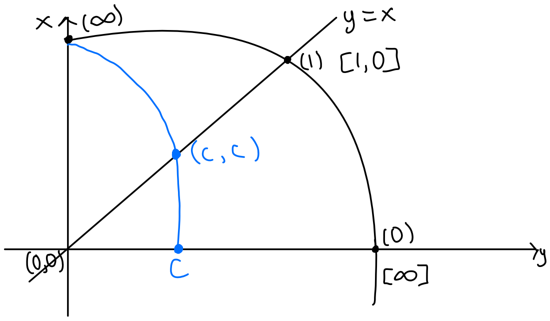
Where $(0,0,1) \cdot [a,b,c] = 0$

& $(1,m,1) \cdot [a,b,c] = 0$

→ $c = 0$

So, $a + mb = 0$

where does $[-m,1,0]$ intersect with $[0,0,1]$?

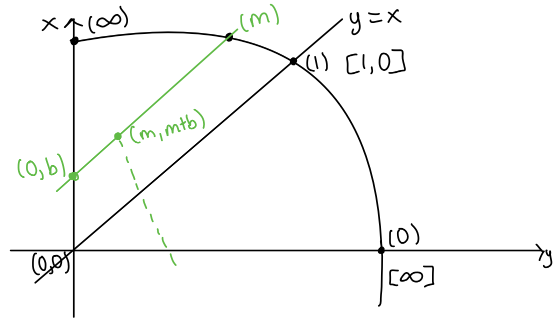


$(c, 0) \to (c, 0, 1)$

$(c, c) \to (c, c, 1)$

$(\infty) \to (0, 1, 0)$

$[c] \to [1, 0, -c]$



$(m) \to (1, m, 0)$

$(0, b) \to (0, b, 1)$

$[m, b] \to [-m, 1, -b]$