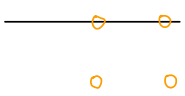
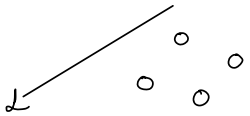
At least three points on every line (in a projective plane)

line :

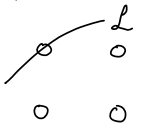


Given a line $\mathcal{L}$, let's consider the possibilities when we look at $\mathcal{L}$ and the four points handed to us by axiom III.

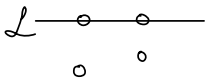
1) $\mathcal{L}$ does not include any of those four points



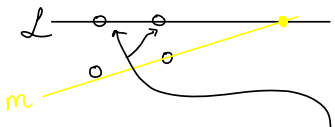
2) $\mathcal{L}$ includes only one of the four points



3) $\mathcal{L}$ includes two of the four points

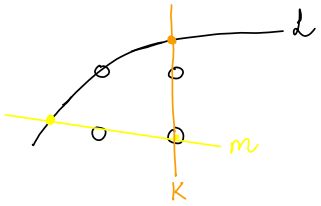


3)

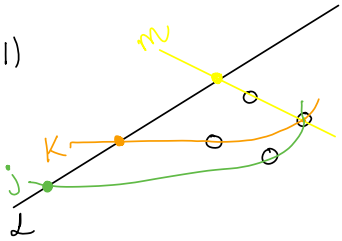


m can't intersect $\mathcal{L}$ in since no three of the four are on the same line.

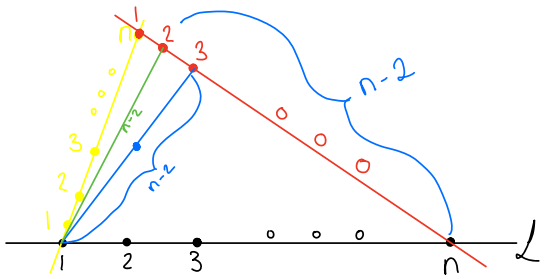
2)



1)

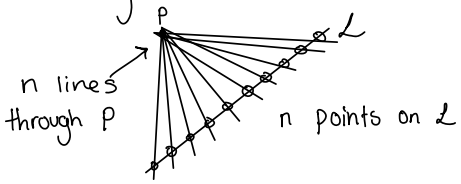


- Suppose that a projective plane has $n \geq 3$ points on every line.
- Q: How many points (and how many lines) are there in such a plane?



$$
\begin{aligned}
&n \\
&+ n-1 \\
&+ n-2 \\
&+ (n-2)(n-2) \\
&= 3n-3+n^2-4n+4 \\
&= n^2-n+1
\end{aligned}
$$

How many lines are there in a plane with n points on every line?



- n points on ℓ each has n lines passing through it:

$n \cdot n - n + 1$

$= n^2 - n + 1$