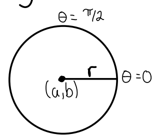
Today: What is the circumference of a circle?




Circumference $= 2\pi r$

$\pi = \frac{C}{d} = \frac{C}{2r}$

- Suppose our circle of radius $r$ has centre $(a,b)$
- Parametrically, points on the circle are given by

$$x = r\cos(\theta) + a$$
$$y = r\sin(\theta) + b$$
$$0 \le \theta \le 2\pi$$

- The length of a parametric curve $x = f(t)$, $y = g(t)$, $c \le t \le d$, is given by

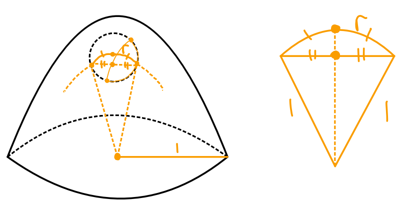
$$\int_c^d \sqrt{(f'(t))^2 + (g'(t))^2}\, dt$$

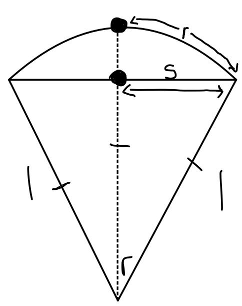
$$\frac{dx}{d\theta} = -r\sin\theta$$

$$\frac{dy}{d\theta} = r\cos\theta$$

$$C = \int_0^{2\pi} \sqrt{(-r\sin\theta)^2 + (r\cos\theta)^2}\, d\theta$$
$$= \int_0^{2\pi} r\underbrace{\sqrt{\sin^2(\theta) + \cos^2(\theta)}}_{=1}\, d\theta$$
$$= r\int_0^{2\pi} 1\, d\theta$$
$$= r\theta\Big|_0^{2\pi}$$
$$= r2\pi - r0$$
$$= 2\pi r$$

- In Euclidean space the circumference of a circle of radius $r$ is $2\pi r$.
- What about in the elliptic plane?



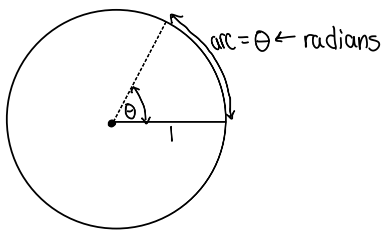

$C = 2\pi s$

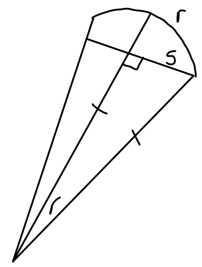
What is $s$?

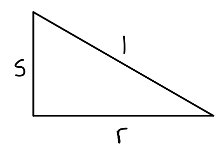
Euclidean: $C = 2\pi r$

Elliptic: $C = 2\pi \sin(r)$

Hyperbolic: $C = 2\pi \sinh(r)$

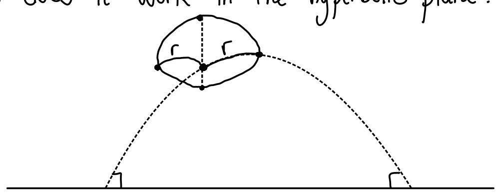



$$\sin(r) = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{s}{1} = s$$

• How does it work in the hyperbolic plane?




By an analogy?

$C = 2\pi$

$$\frac{\sin(\alpha)}{a} = \frac{\sin(\beta)}{b} = \dots$$

$$\frac{\sin(\alpha)}{\sin(a)} = \frac{\sin(\beta)}{\sin(b)} = \dots$$

$$\frac{\sin(\alpha)}{\sinh(a)} = \frac{\sin(\beta)}{\sinh(b)} = \dots$$