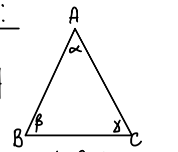
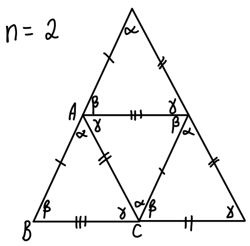
Legendre's Second Theorem

- If you have one triangle with $\alpha+\beta+\gamma=\pi$, then all do this.

Proof:

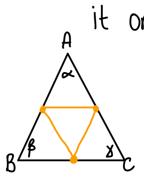
$\exists$  $\Rightarrow$ Post. V'

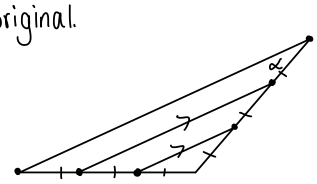
$\alpha+\beta+\gamma=\pi$



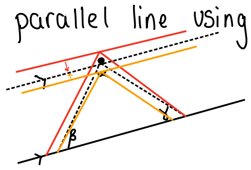
Similarly, we can make similar copies of ΔABC that are n times larger for each $n>1$.

- Suppose we want a scaled version of ΔABC, scaled by a factor of $1/2$. How do we get it?
- Shrink the $n=2$ double-size one and it will still be similar to the original & then lay it on the original.





- Given that we can expand by n and shrink by $1/n$, we can scale ΔABC by any positive rational quantity a/b.
- Now what?
- Get parallel line using the easy part of the Z-theorem.



- Fitting in a saled version of ΔABC requires arbitrary scaling by real numbers.
- Say we need to scale by some $r \in \mathbb{R}$.
- If r is rational, we know we can do it.
- If r is not rational, we approximate r from above and below by sequences of rationals converging to r and build scaled triangle (with parallel lines) accordingly.
- We get the picture we want as the common limit of both sequences.

- Say we want $\sqrt{2}$, and we can get any rational we like. What can we do?
  $1, 1.4, 1.41, 1.412, \ldots$ has limit $\sqrt{2}$.