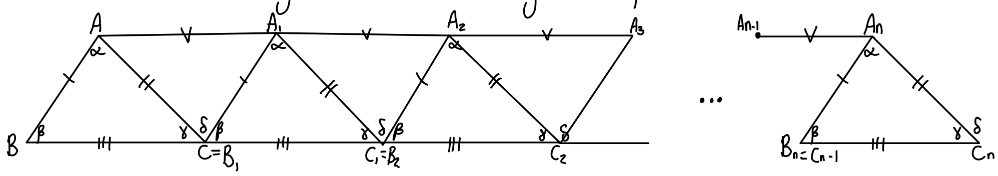
Legendre's First Theorem (in geometry)

- In the presence of Postulates I - IV, then the assumption that there are no parallels leads to a contradiction.
- Actually, the sum of the internal triangle cannot exceed two right angles.

Proof: Assume there is a triangle whose internal angles add up to more than π.



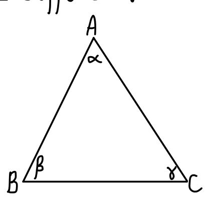
$\alpha + \beta + \gamma > \pi$ Since $\delta + \beta + \gamma = \pi$ we have $\delta < \alpha$.

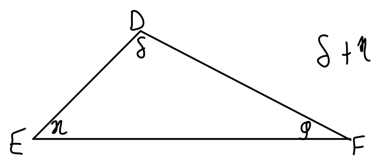
- Since $\delta < \alpha$ and $AB = B_1A_1$ and $AC = AC$ we have that $AA_1 < BC$.
- If we continue to extend BC and building copies of ΔABC on the extension, we will eventually get the path $BAA_1A_2 \dots A_nC_n$ will be shorter than the path $BCC_1C_2 \dots C_n$.
- This means that the straight line BC_n is <u>not</u> the shortest path from B to C_n, contradicting the fact that straight lines are shortest paths.

Legendre's Second Theorem

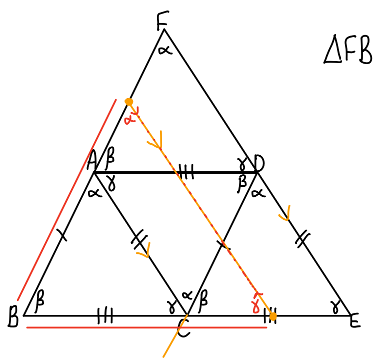
- If there is a single triangle with a sum of interior angles equal to two right angles [less than two right angles], then every triangle has a sum of interior angles equal to [resp. less than] two right angles.

Proof: Suppose we have ΔABC such that $\alpha + \beta + \gamma = \pi$.



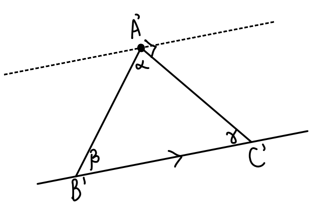


$\delta + \eta + \varphi = ?$



$\Delta FBE \sim \Delta ABC$

- We'll try to show that the existance of ΔABC with $\alpha + \beta + \gamma = \pi$ is enough to prove Playfair's Postulate (Post. V')



Idea: Put a scaled copy of ΔABC with BC on the line and A' at the point