Today, working up to Legendre's Theorems related to Post. V.

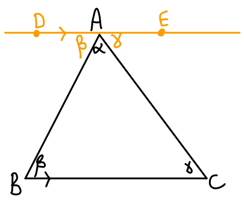
First,

<u>Prop:</u> Postulate V $\Leftrightarrow$ the sum of the interior angles of a triangle is π.

$V \Leftrightarrow \alpha+\beta+\gamma=\pi$

<u>Proof:</u> $\boxed{\Rightarrow}$

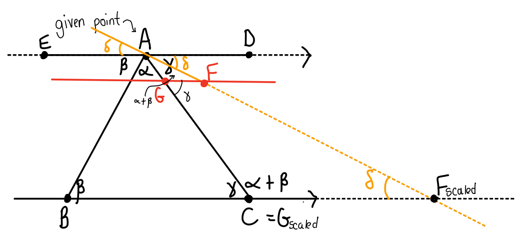
· Assume Post. V



· By Post. V there is an unique line through A parallel to BC

· By the Z-theorem (I-29), $\angle DAB=\beta$ and $\angle EAC=\gamma$. Thus $\angle DAE=\pi=\beta+\alpha+\gamma$.

$\boxed{\Leftarrow}$



· Assume that for any triangle $\alpha+\beta+\gamma=\pi$

· [To show: Post V']

· Pick B and C on the given line and make ΔABC

· Pick D such that $\angle DAC=\gamma$ and E such that $\angle EAB=\beta$

· E, A, D are collinear because $\angle EAD=\beta+\alpha+\gamma=\pi$

· We need to show it is parallel to BC.

· By the Z-theorem (I-28) since $\angle EAB=\angle ABC$, EA is parallel to BC.

· Why is ED the <u>unique</u> line through A parallel to BC?

· Suppose another line through A passes between D and C. Pick a point F on this line (on the same side of BC as D is)

· Draw a line through F, intersecting AC at a point G s.t. $\angle AGF=\alpha+\beta$

· Then by the Z-theorem (I-28), GF is parallel to BC.

· Scale ΔAGF by $\frac{|AC|}{|AG|}$ ... and place the scaled version with AG on AC

· Then GF lies along BC and so AF meets BC at the new scaled F

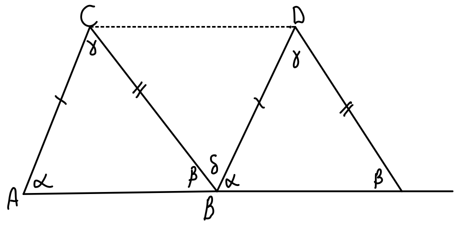
<u>Legendre's First Theorem</u>

Assume Postulate I - IV (and S and A and ...) hold, then the sum of the interior angles of a triangle cannot exceed two right angles.

[The elliptic plane is impossible with an unmodified Post. II.]

<u>Proof:</u>

· Assume by way of contradiction that ΔABC has $\alpha+\beta+\gamma>\pi$



$\alpha+\beta+\delta=\pi<\alpha+\beta+\gamma \Rightarrow \delta<\gamma$

so, $|CD|<|AB|$