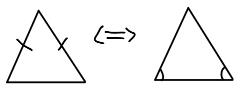
Some more "neutral" geometry (Applicable in the Euclidean, elliptic, and hyperbolic planes).

<u>I-5 and I-6</u>

- The base angles of a triangle are equal iff the corresponding sides are equal.




<u>I-7</u>

- Side-Side-Side congruence criterion

<u>I-8</u>

- Angle-Side-Angle congruence criterion

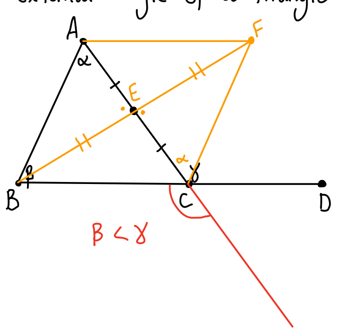
<u>I-9</u>

- A lot of basic constructions (bisecting angles, bisecting lines, constructing right angles, ...)

⋮

<u>I-16</u>

- Any external angle of a triangle exceeds either of the opposite interior angles.




<u>To Show</u>: $\gamma > \alpha$ and $\gamma > \beta$

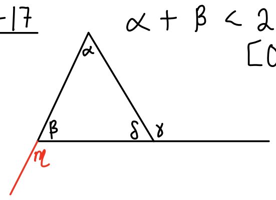
- $\Delta AEB \cong \Delta CEF$ by S-A-S
- So, $\angle ECF = \angle EAB = \alpha$
- So, $\gamma = \angle ACD$

$\quad = \angle ACF\ (\alpha) + \angle FCD$

$\therefore \alpha < \gamma$

<u>I-17</u> $\alpha + \beta < 2\llcorner$

[Obviously, $\alpha < \gamma < 2\llcorner$ and $\beta < \gamma < 2\llcorner$]




$\delta + \gamma = 2\llcorner$

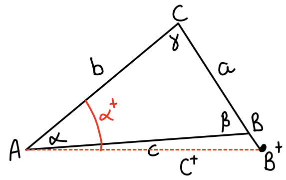
$\Rightarrow \delta + \alpha < 2\llcorner$

and $\delta + \beta < 2\llcorner$

and $\alpha + \beta < \eta + \beta = 2\llcorner$

## I-18 and I-19

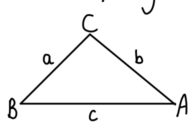
- In any triangle greater sides subtend greater angles and vice versa.




$\beta > \alpha$
$a < b$

## I-20

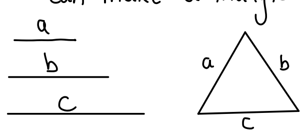
- The sum of any two sides of a triangle exceed the third side.



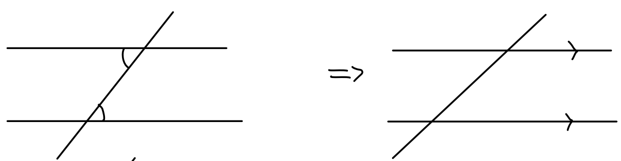

$BA = C$ has to be less than $BCA = a + b$

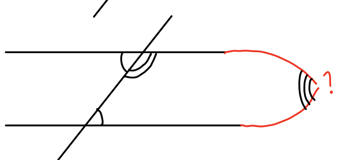
## I-22

- Given three line segments such that the sum of any two exceed the third, you can make a triangle with sides of these lengths.



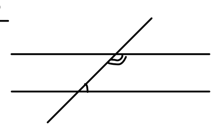

## I-27

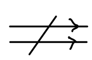


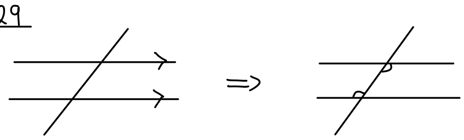

- If the alternate interior angles are equal, why are the lines parallel?
- Suppose the lines actually met at some point.
- Then $\angle + \angle = 2\llcorner$ but I-17 tells us $\angle + \angle < 2\llcorner \Rightarrow \therefore$ contradiction

## I-28



$\angle + \angle = 2\llcorner \Rightarrow$ 

## I-29




- This requires Post V ...