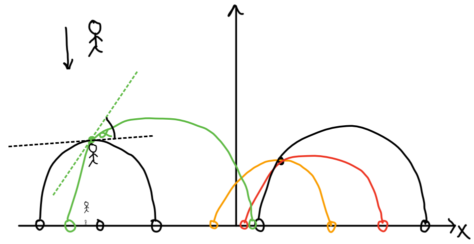
The Hyperbolic Plane (via the Poincaré half-plane model)



- The points are all the points in the Cartesian plane <u>above</u> the x-axis.
- The lines are semi-circles centred on the x-axis
- Angles between "lines" are the angles between (the tangent lines to) the semicircles

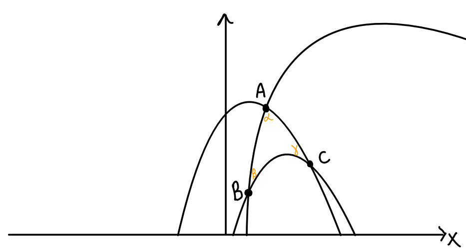
- Hyperbolic distance in this model

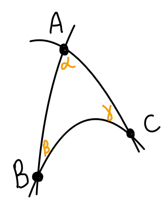
  $A(x_1, y_1)$

  $B(x_2, y_2)$

  If $x_1 = x_2$, the distance from A to B is $\left|\ln\left(\frac{y_1}{y_2}\right)\right|$.

  If $x_1 = x_2$, and $(c,0)$ is the centre of the "line" joining the points and has radius, then the distance from A to B is $\left|\ln\left(\frac{(x_1-c-r)y_2}{(x_2-c-r)y_1}\right)\right|$.





- In the elliptic plane the sum of the interior angles of a triangle is $\alpha+\beta+\gamma > \pi$.
- In the Euclidean plane $\alpha+\beta+\gamma = \pi$.
- In the hyperbolic plane $\alpha+\beta+\gamma < \pi$.

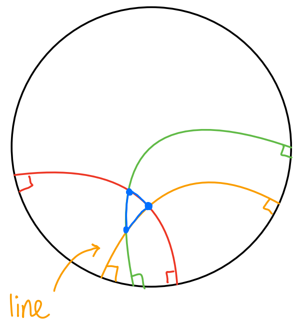
Hyperbolic cosine laws:

- $\cosh(a) = \cosh(b)\cosh(c) - \cos(\alpha)\sinh(b)\sinh(c)$
- $\cosh(x) = \dfrac{e^x + e^{-x}}{2}$
- $\sinh(x) = \dfrac{e^x - e^{-x}}{2}$
- $\cos(\alpha) = \cosh(a)\sin(\beta)\sin(\gamma) - \cos(\beta)\cos(\gamma)$

Sine Law:

- $\dfrac{\sin(\alpha)}{\sinh(a)} = \dfrac{\sin(\beta)}{\sinh(b)} = \dfrac{\sin(\gamma)}{\sinh(c)}$

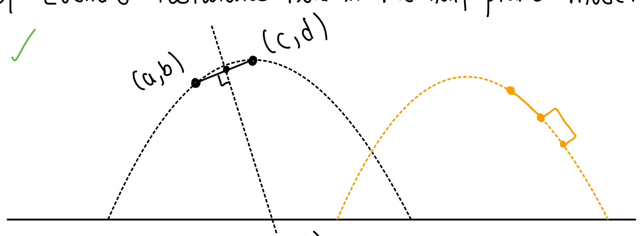
- Area of $\Delta ABC = \pi - (\alpha+\beta+\gamma)$



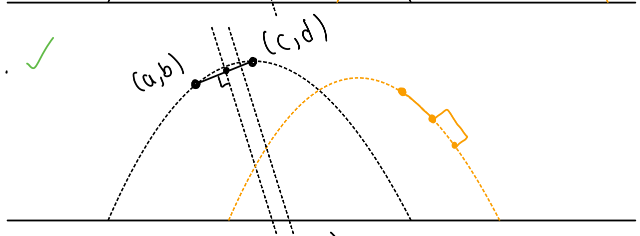
Points: points inside a unit circle

Which of Euclid's Postulates hold in the half-plane model?

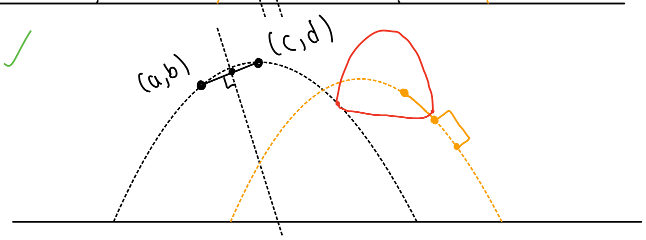
1. ✓



2. ✓



3. ✓



4. ✓

5. ✗

Post. S ✓

Post. A ✓