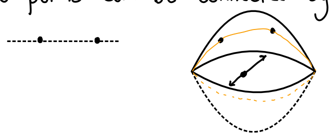
Back to the elliptic plane (via the antipodal sphere model)

<u>Shpere</u> <u>vs.</u> <u>Antipodal Sphere model (of the elliptic plane)</u>

- triangles & their areas work pretty much the same way
- max distance between points on a sphere (with $r=1$) is π, on the antipodal sphere model it is $\pi/2$.
- great circles intersect twice, their counterparts do so once

Which of Euclid's Postulates does the antipodal sphere model satisfy?

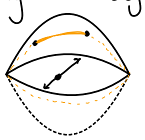
I. Any two points can be connected by a line (segment)



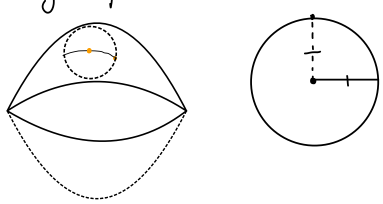
YES

II. Given any line (segment), it can be extended as far as you like in either direction.

NO (unless you allow overlap)

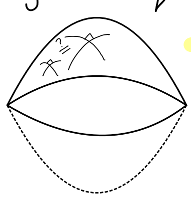


III. Given a point and a line segment attached to it, you can draw a circle with the given point as the centre and the given line segment as the radius.



YES

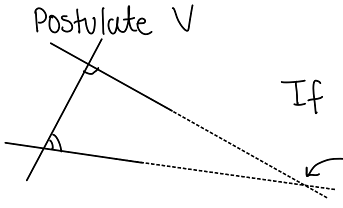
IV. All right angles are equal



YES

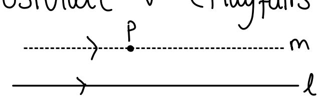
V.

Postulate V



If $\measuredangle + \measuredangle < 2\llcorner$, then

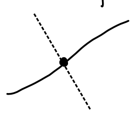
Postulate V' (Playfairs Axiom)



→ Given a point P and line ℓ not through P, there is a unique line m through P parallel to ℓ.

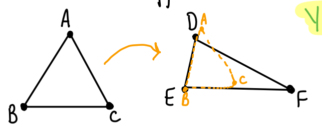
<u>NO</u> - any two "lines" intersect

Postulate S [separation]



YES

Postulate A [application]

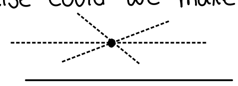


YES

Non-Euclidean geometry is denying Postulate V and keeping as much as you can of the others:

Elliptic plane: Deny V' by not allowing parallels. Price: modifying Post. II because lines come back to themselves.

How else could we make Post V' fail?



Violate the uniqueness condition instead

Denying uniqueness of the parallel line in V' gives the <u>hyperbolic plane.</u>
(Keep <u>all</u> of Postulates I - IV unchanged)