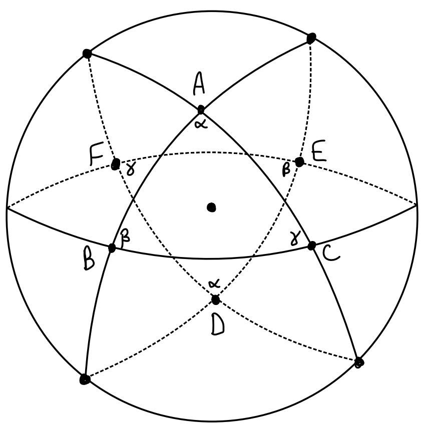
Spherical triangles (applicable to the "elliptic" plane)

$r = 1$

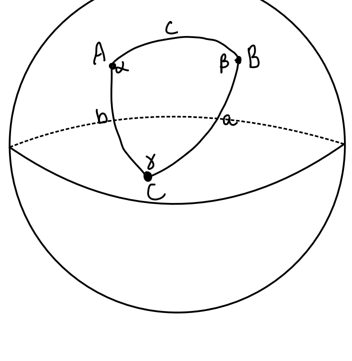


- Given ΔABC, let ΔDEF be it's symmetric opposite.
- Consider the following lunes

lune	Area
ABDC	2α
AFDE	2α
BAEC	2β
BFED	2β
CAFB	2γ
CDFE	2γ

- Area of a hemisphere of a sphere of radius 1 is $\frac{4\pi 1^2}{2} = 2\pi$
- The hemisphere above the great circle that BC is part of has area 2π.
- It can be decomposed into $\Delta ABC + \Delta AFE + \Delta ABF + \Delta AEC$
- $\text{area}(\Delta ABC + \Delta AFE) = \text{area}(\Delta ABC + \Delta BCD)$
  $= \text{area}(\text{lune } ABDC)$
  $= 2\alpha$
- $\text{area}(\Delta ABC + \Delta ABF) = \text{area}(\text{lune } CBFA)$
  $= 2\gamma$
- $\text{area}(\Delta ABC + \Delta AEC) = \text{area}(\text{lune } BAEC)$
  $= 2\beta$

  $\therefore$ area of hemisphere $+ 2\,\text{area}(\Delta ABC) = 2\alpha + 2\gamma + 2\beta$
- $2\alpha + 2\beta + 2\gamma = 2\pi + 2\text{area}(\Delta ABC)$
  $\Rightarrow \text{area}(\Delta ABC) = \alpha + \beta + \gamma - \pi$
- As long as the triangle fits in <u>one</u> hemisphere we must have $\alpha + \beta + \gamma > \pi$
- On the other hand the sum $\alpha + \beta + \gamma$ must be bounded above by $\alpha + \beta + \gamma < 3\pi$



- In ΔABC, let α, β, γ be the interior angles at A, B, C resp. and a, b, c be the lengths of the sides opposite to A, B, C resp.

- In Euclidean space, given a ΔABC with angles & side lengths labelled similarly we have:
  (1) The Law of cosine: $c^2 = a^2 + b^2 - 2ab\cos(\gamma)$
  (2) The Law of Sines: $\dfrac{c}{\sin(\gamma)} = \dfrac{B}{\sin(\beta)} = \dfrac{a}{\sin(\alpha)}$

- In Spherical trigonometry we have two laws of cosines:
  (1) $\cos(a) = \cos(b)\cos(c) + \cos(\alpha)\sin(b)\sin(c)$
  (2) $\cos(\alpha) = \cos(a)\sin(\beta)\sin(\gamma) - \cos(\beta)\cos(\gamma)$

- In Spherical trigonometry we have two laws of Sines:
  $\dfrac{\sin(\alpha)}{\sin(a)} = \dfrac{\sin(\beta)}{\sin(b)} = \dfrac{\sin(\gamma)}{\sin(c)}$

- Q: Which congruence criteria for triangles work on the (unit) sphere?
  - ✓ Angle-Angle-Angle congruence works because we can recover side length from the angle using the cosine & sine laws.
  - ✓ Side-Side-Side congruence works too for a similar reason
  - ✓ Side-Angle-Side
  - ✓ Angle-Side-Angle
  - ✗ Angle-Angle-Side (need more information)
    ↳ $\dfrac{\sin(\alpha)}{\sin(a)} = \dfrac{\sin(\beta)}{\sin(b)} = \dfrac{\sin(\gamma)}{\sin(c)}$
  - ✗ Angle-Side-Side (need more information)