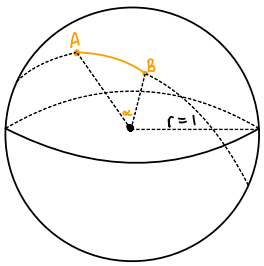
Spherical geometry (by way of setting up elliptic plane geometry)

Sphere of radius 1

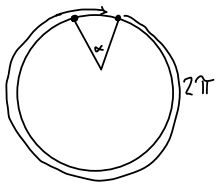


Basics:

- length of a great circle $= 2\pi r = 2\pi$
- Surface area of the sphere $= 4\pi r^2 = 4\pi$
- Volume $= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi$

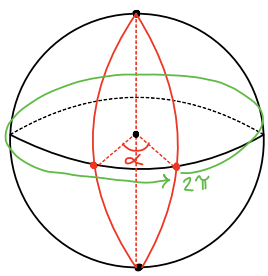
- What's the distance between A and B (along the great circle connecting them)?

dist(A,B) $= \alpha$



- radians measure arclength of a unit circle

- Areas on the sphere

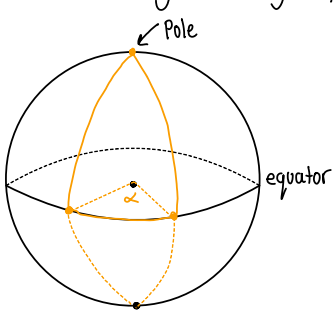


- A segment like this (joining two antipodal points by great circles and looking at the region between) is called a "lune".

- Area of this lune?
  - It's the proportion of α to 2π applied to the area of the sphere 4π.

$$A = \frac{\alpha}{2\pi} \cdot 4\pi$$
$$= 2\alpha$$

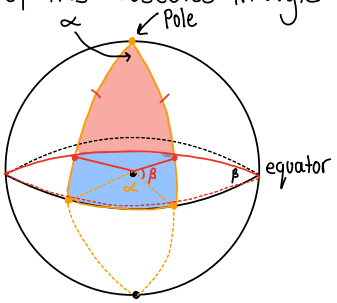
- Area of a triangle including a pole and two equatorial points?

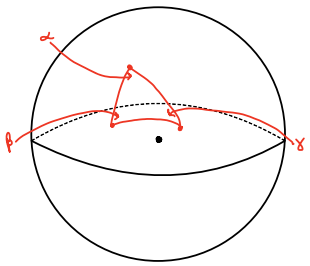


Area $= \frac{1}{2}$ of area of lune with central angle α

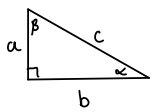
$$= \frac{1}{2} 2\alpha$$
$$= \alpha$$

- Area of this isosceles triangle?





$A = \alpha + \beta + \gamma - \pi$ ← in terms of angles

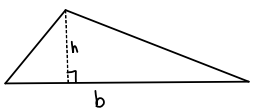


$a^2 + b^2 = c^2$

$\sin(\alpha) = \frac{a}{c}$

$\cos(\alpha) = \frac{b}{c}$

⋮



$A = \frac{1}{2}ab$

$\Rightarrow A = \frac{1}{2}bh$ ← in terms of side length