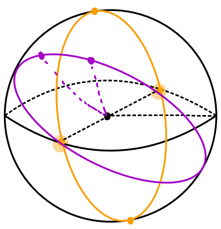
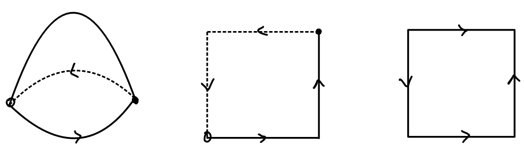
Today: non-Euclidean geometry, starting with restoring measurement to the real projective plane

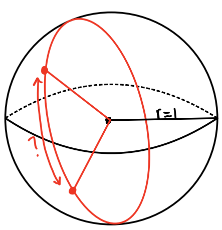
- We got the real projective plane by starting with the Euclidean plane, throwing away all measurement, and then adding the line at infinity.
- This is hard to bring measurement back to since the line at infinity is special in terms of distances and angles.
- We'll use a derivation of the linear algebra way of constructing this plane instead.
  - lines = planes through the origin in $\mathbb{R}^3$ (ie 2-D subspaces in $\mathbb{R}^3$)
  - points = lines through the origin in $\mathbb{R}^3$ (ie 1-D subspaces of $\mathbb{R}^3$)
  - incidence = inclusion
- We can get angles between lines back by using the angles between the corresponding planes.
- How do we get distances between points?
- Our version of the real projective plane with measurement ("elliptic plane") will be the geometry of points and great circles ("lines") of a sphere with antipodal points "identified" (ie being the same point)

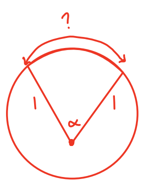


- great circles intersect at two points



- We can still measure angles using the angles between the planes, which are also the angles between the great circles when they cross. We can measure distance between two points along the great circle joining them. (that comes from the plane defined by those two points and the centre of the sphere)





- If we measure angles in radians, and $r=1$, then the arc subtended by a central angle of α is α.