Incidence (and axioms):

Informally: A point and a line are incident if "the point is on the line" or "the line passes through the point."

In general: An <u>incidence</u> <u>structure</u> is a triple $(\mathcal{P}, \mathcal{L}, I)$ where $\mathcal{P}$ is a (nonempty) set of "points", $\mathcal{L}$ is a (non-empty) set of "lines", and I is a relation between "points" and "lines".

- If $P \in \mathcal{P}$ and $\ell \in \mathcal{L}$, then $P\, I\, \ell$. ("P is incident with ℓ") means that P is on ℓ.

Examples:

(1) The cartesian plane

$\mathcal{P} = \{(x,y) \mid x,y \in \mathbb{R}\}$

$\mathcal{L} = \{y = mx+b \mid m,b \in \mathbb{R}\}$ (not both 0)

$\cup \{x = c \mid c \in \mathbb{R}\}$

What is I here?

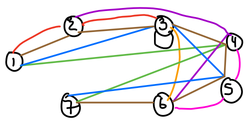
$P\, I\, \ell$ if the cooridnates of P satisfy the equation of ℓ.

(2) $\mathcal{P} = \{1,2,3,4,5,6,7\}$

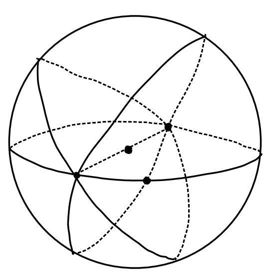
$\mathcal{L} = \{\{1,2,3\}, \{4,5,6\}, \{1,4,7\}, \{2,4,6\}, \{1,3,5,7\}, \{3,6\}, \{3\}, \{1,2,3,4,5,6,7\}\}$

What is I here?

$P\, I\, \ell$ exactly when $P \in \ell$



(3)



$\mathcal{P} = \{$points on the surface of a sphere$\}$

$\mathcal{L} = \{$great circles on a sphere$\}$

I is inclusion.

<u>Def'n:</u> A <u>configuration</u> is an incidence structure with the following properties:

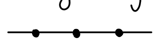
(i) Any two points are incident with at most one line.

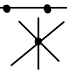
(ii) Any two lines are incident with at most one point.

Examples:

(1) Euclidean plane is a configuration.

~~(2)~~ The geometry of great circles on a sphere is <u>not</u> a configuration.

(3) 

(4) 

(5) 

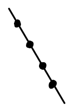
~~(6)~~

Def'n: An affine plane is a configuration satisfying the following axiom is:

(A I) Any two points are incident with a (unique) line.

(A II) [Playfair's Axiom] Given a point P and a line ℓ with $P \not I \ell$, there is a (unique) line through P which does not meet ℓ at any point.

We need one more axiom:



We don't want this to be a plane.

(A III) There exist three points which are not all on the same line.

Def'n: A projective plane is a configuration such that:

(I) Any two points are incident with a (unique) line.

(II) Any two lines are incident with a (unique) point.

(III) There exist four points such that no three are on the same line

e.g.