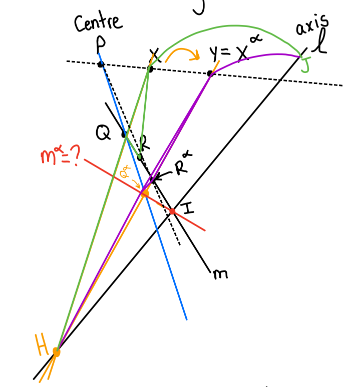
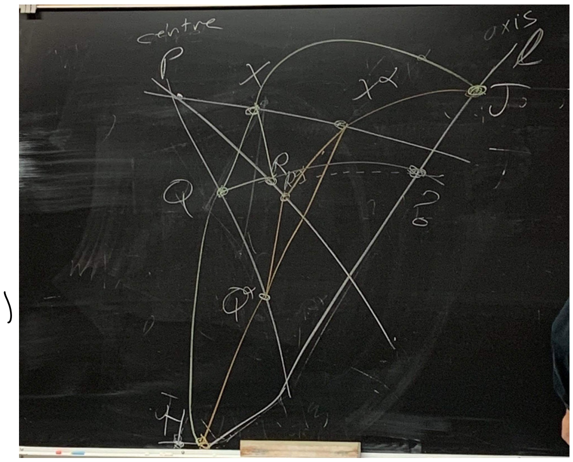
Back to showing the α we defined preserves incidence.

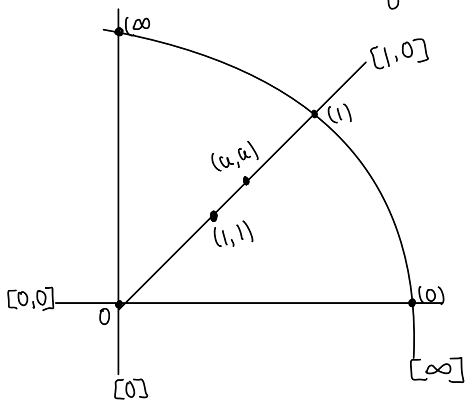


"Preserves incidence" means $Q I m \iff Q^\alpha I m^\alpha$



- Suppose R is any other point on m (other than $m \cap \ell$)
- We need to show that R, Q, I are collinear.
- Consider ΔXQR
- $X \xrightarrow{\alpha} Y = X^\alpha$
- $Q \xrightarrow{\alpha} Q^\alpha$
  $= PQ \cap YH$ where $H = XQ \cap \ell$
- $R \xrightarrow{\alpha} R^\alpha = PR \cap YJ$ where $J = XR \cap \ell$
- This gives $\Delta X^\alpha Q^\alpha R^\alpha$
- To preserve incidence we need that QR intersects ℓ at the same point that $Q^\alpha R^\alpha$ does, which happens by the (P,ℓ)-Desargues' Thm. //
  ie. we're moving points around by α in a way that preserves lines as sets of points.

- A plane is (p,ℓ)-transitive if for any x, y collinear with P (and $\neq P$ and $\not\mathrel{I}\, \ell$) there is a (P,ℓ)-central collineation α s.t. $y = x^\alpha$.
- So on way to say what we've done is that a plane is (P,ℓ)-transitive iff the (P,ℓ)-Desargues' Thm holds.
- We'll circle back to the algebra coordinates of the plane...



- $(x,y) \; I \; [m,b] \iff y = T[m,x,b]$ $T \to$ "ternary ring"
- If we define $+$ by $a+b = T(1,a,b)$
  & $\bullet$ by $ab = T(a,b,0)$
  is $T(m,x,b) = (m \cdot x) + b$?
- If $T(m,x,b) = m \cdot x + b$, then T is "linear".
- We'll look (a bit) at situations involving $(0,0), (0), (\infty)$ &/or $[\infty], [0,0], [0]$ for which (P,ℓ)-Desargues' Thm $\Rightarrow$ linearity.