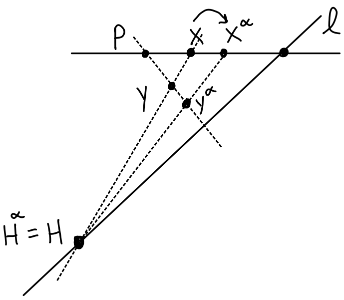
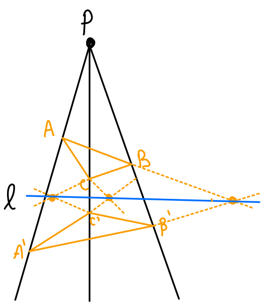
(P,ℓ) - central (axial) collineation α is one that has $m^\alpha = m$ for every line m with $P \,I\, m$ & $Q^\alpha = Q$, for every point Q with $Q \,I\, \ell$.

↑ Point ↑ Line

(Could have $P\,I\,\ell$ or not. - A7)

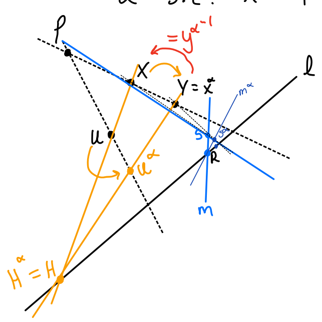


once you know how a (P,ℓ) - central collineation moves <u>one</u> point you know everything.



(P,ℓ) - Desargues' Thm: If AA', BB', CC' are all incident with P and $AC \cap A'C'$ and $BC \cap B'C'$ are incident with ℓ then $AB \cap A'B'$ is also incident with ℓ.

<u>Thm:</u> Suppose the (P,ℓ)-Desargues Thm holds and X & Y are points not on ℓ and collinear with P (but not equal to P). Then there is a (P,ℓ)-central collineation α s.t. $X^\alpha = Y$



<u>Proof:</u> (Reader's Digest abbreviated edition)

We define α as follows:

$P^\alpha = P$, $Q^\alpha = Q$ for all $Q \,I\, \ell$, and $X^\alpha = Y$.

If u is a point not on ℓ or on the line XY, then $u^\alpha = Pu \cap X^\alpha H$ where $H = Xu \cap \ell$.

We can move other points on XY by looking how some u off that line moves

This defines how α acts on the <u>points</u> of the plane.

(1) We defined it to take points to points.

(2) It's 1-1 because α has an inverse ⤳ α has an inverse because the definition is reversible. (α^{-1} moves Y to X etc.)

(3) It's onto because ...

How does α move lines?

Given a line m, what is m^α?

($\ell^\alpha = \ell$) If $m \neq \ell$, then do the following: let $R = m \cap \ell$.

axis

Let $S \neq R$ be another point on m

Then $m^\alpha = RS^\alpha$

α takes lines to lines (by def'n) & is 1-1 & onto because α is invertible ...

For α to be a collineation we need to check that α preserves incidence.

Why?