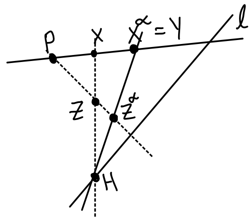
Today: (P, ℓ) - Desargues' Thm

$\Rightarrow$ We can move points pretty freely by (P,ℓ)-collineations



- If we define α by taking $X^\alpha = Y$ and determining how all other points move from that, we need to check this actually is a collineation (with centre P & axis ℓ)
- What do we need to prove about α?

(1) α is 1-1 & onto on the points:

By its definition, it does have points to points.
(It is a function, by it's definition it assigns one point to each input point)
α is 1-1 & onto (for the points) because it has an inverse
α^{-1} is the function that moves Y to X and is defined in the same way as α.
Since the definition of α is effectively reversible, α^{-1} exists and so α is 1-1 & onto.

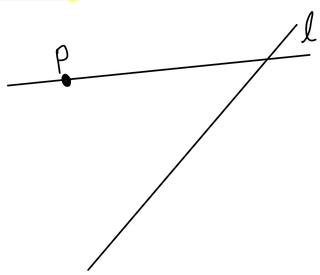
(2) α moves lines to lines:

(because we defined α to move points in a way that moved lines...)
& is reversible because it is on points and so it is 1-1 & onto (on the lines)

(3) α preserves incidence. Why?

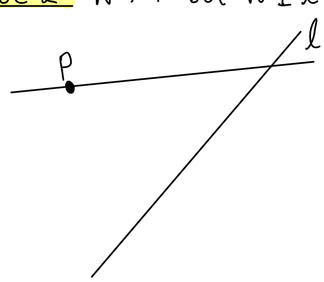
We need to check that for any point W and line m, $W I m \Longleftrightarrow W^\alpha I m^\alpha$

$\rightarrow$ Case 1: $W = P$ & $m = \ell$



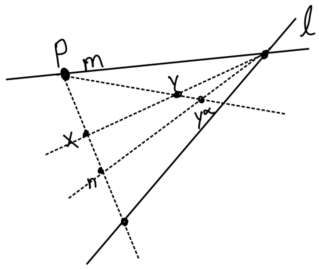
By the definition of α $\quad W I m \Longleftrightarrow P I \ell$
$\Longleftrightarrow P^\alpha I \ell^\alpha$ (as $P^\alpha = P$ & $\ell^\alpha = \ell$ by the def' of α)
$\Longleftrightarrow W^\alpha I m^\alpha$

$\rightarrow$ Case 2: $W \neq P$ but $W I \ell$ $[m = \ell]$



$W I m \Longleftrightarrow W I \ell$
$\Longleftrightarrow W^\alpha I \ell^\alpha$ (since $W^\alpha = W$ & $\ell^\alpha = \ell$)
$\Longleftrightarrow W^\alpha I m^\alpha$

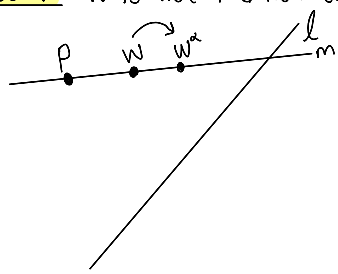
→ **Case 3:** $W = P$ and $P\, I\, m$



If $W=P$ and m is a line s.t. $P\, I\, m$, then $m \cap \ell$ is not moved by α and every other point X on m gets moved to a point X^α also on m.

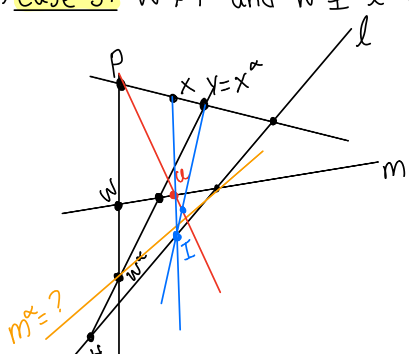
Thus, $m^\alpha = m$, $W^\alpha\, I\, m \iff P\, I\, m$
$\iff P^\alpha\, I\, m^\alpha$

→ **Case 4:** W is not P & not on ℓ and $m \neq \ell$ but $P\, I\, m$



By the definition of α, W^α will be a point on m.
So, W^α will be on the line PW
If $m = PW$ (ie $W\, I\, m$) then $W^\alpha\, I\, m^\alpha$
If $m \neq PW$ then $m^\alpha = m$, and W^α is on PW so $W^\alpha\, \not{I}\, m^\alpha$

→ **Case 5:** $W \neq P$ and $W\, \not{I}\, \ell$ and m is not passing through P and $m \neq \ell$



If $W\, I\, m$

Come back after reading week.