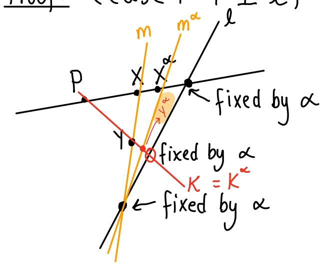
Today: moving points around with central collineations

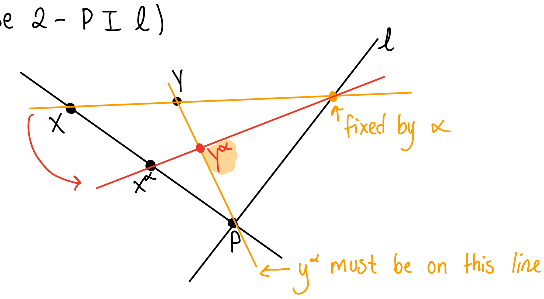
Once you know how a (P, ℓ)-central collineation moves one point X (where $X \neq P$ and $X \not{I} \ell$)
you can determine how it moves <u>all</u> the others.

<u>Proof:</u> (case 1 - $P \not{I} \ell$)

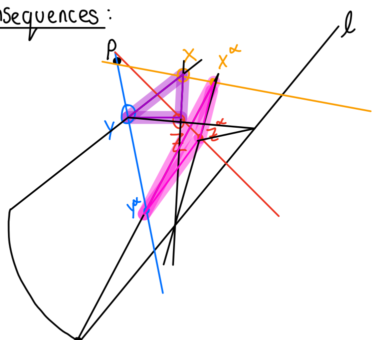


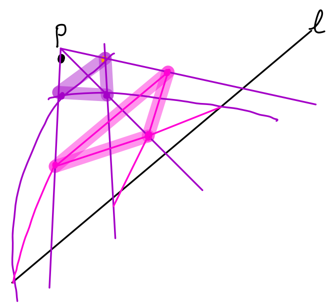
Given X^α is known, where does Y go?

(case 2 - $P \, I \, \ell$)



<u>Consequences</u>:

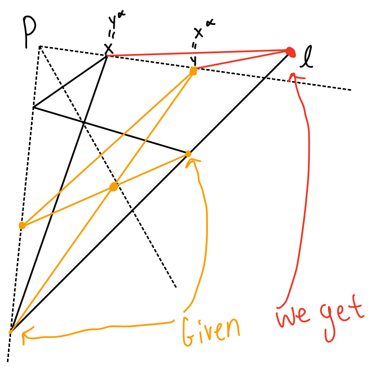




- X, Y, Z are <u>not</u> collinear (none are P & none are on ℓ)
- α a (P, ℓ) — Central collineation
- α must take any triangle to one in perspective to the first from P as well as ℓ

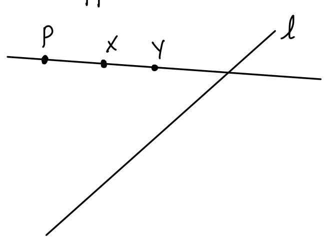
<u>Desargues' Thm</u> (projective version)
(true in Euclidean geometry)
- Two triangles are in perspective from a point iff they are in perspective from a line

<u>(P, ℓ) - Desargues' Thm</u>
(where P is a point and ℓ is a line)
Two triangles that are in perspective from P and have two pairs of corresponding sides meet on ℓ have the third pair of corresponding sides meet on ℓ too.

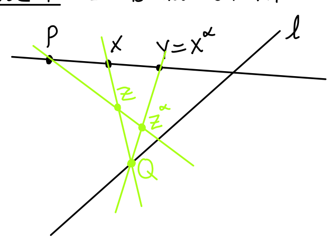


<u>Thm</u>: Given (P, ℓ) in a projective plane, if the (P,ℓ)-Desargues Theorem holds and X and Y are any points ($\neq P$ and $\not\in \ell$) that are collinear with, then there is a (P,ℓ) central collineation that moves X to Y.

<u>Proof:</u> Suppose we have the hypotheses hold.

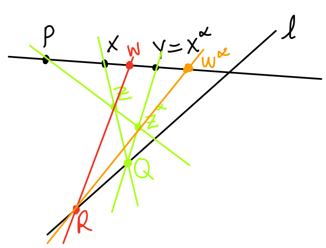


- We need to define α for any point Z (not $= P, X, Y$ or not on ℓ)
- <u>Case 1:</u> Z is not on XY



- let $Q = XZ \cap \ell$ & define $Z^\alpha = PZ \cap QY$

<u>Case 2:</u> W is a point on PX that is not X or P or on ℓ.
What must W^α be?



- let Z be a point (like in case 1 not on PX or on ℓ). we know Z^α by case 1. Let $WZ \cap \ell = R$
- Let $W^\alpha = RZ^\alpha \cap PX$

This defines α

We need to verify that α is a (P,ℓ)-central collineation.