$T(m,x,b)$
$? = m\cdot x + b$

We'll be looking at collineations of projective planes

Def'n: A collineation of a projective plane $(\mathcal{P}, \mathcal{L}, I)$ is a function.
$\alpha = \mathcal{P} \xrightarrow[\text{onto}]{1\text{-}1} \mathcal{P}$
& $\alpha = \mathcal{L} \xrightarrow[\text{onto}]{1\text{-}1} \mathcal{L}$
that preserves incidence.

$P\, I\, \ell \iff \alpha(P)\, I\, \alpha(\ell)$

Notation: $\alpha(P)$ is usually denoted by P^{α} & $\alpha(\ell)$ by ℓ^{α}

composition

$\alpha(\beta(P)) = P^{\beta\alpha} = P^{\alpha\circ\beta}$

Trivial example:
The identity function i $P^{i} = P$ for all points P & $\ell^{i} = \ell$ for all points ℓ

Less trivial example:
Recall that we used projective coordinates as one way to build the real projective plane.
Points: (a,b,c) $a,b,c \in \mathbb{R}$ (not all 0)
Lines: $[m,n,k]$ $m,n,k \in \mathbb{R}$ (not all 0)
In both cases multiplying all the coordinates by $\lambda \neq 0$ gives the same point or line.
Incidence: $(a,b,c)\, I\, [m,n,k] \iff am+bn+ck=0$
Define a collineation γ on this structure by $(a,b,c)^{\gamma} = (b,c,a)$ and $[m,n,k]^{\gamma} = [n,k,m]$
Obviously, this is 1-1 and onto
Incidence is preserved:
$(a,b,c)\, I\, [m,n,k]$
$\iff am+bn+ck=0$
$\iff bn+ck+am=0$
$\iff (b,c,a)\, I\, [n,k,m]$
$\iff (a,b,c)^{\gamma}\, I\, [m,n,k]^{\gamma}$

Suppose we have a projective plane $(P, \mathcal{L}, I)$ and γ and δ are collineations of it.
Then $\gamma\delta = \delta\circ\gamma$ is also a collineation of the plane, as is γ^{-1}.

Proof:
(1) γ^{-1} is a collineation
γ^{-1} is one to one and onto (for both points and lines) because γ is 1-1 and onto
γ^{-1} preserves incidence:
$P\, I\, \ell \iff P^{\gamma^{-1}\gamma}\, I\, \ell^{\gamma^{-1}\gamma}$
$\iff (P^{\gamma^{-1}})^{\gamma}\, I\, (\ell^{\gamma^{-1}})^{\gamma}$
$\iff P^{\gamma^{-1}}\, I\, \ell^{\gamma^{-1}}$

We have $P^{\gamma}\, I\, \ell^{\gamma}$
$\iff P\, I\, \ell$ for all P & ℓ

• $\gamma\gamma^{-1} \iff \gamma\circ\gamma^{-1}$ is the identity collineation.

(2) We need to show that if γ & δ are collineations of $(\mathcal{P}, \mathcal{L}, I)$, then so is $\gamma\delta = \delta\circ\gamma$.
• $\gamma\delta$ is 1-1 because both γ and δ are 1-1.

- $\gamma\delta$ is onto because γ and δ are onto.
- $P \,I\, \ell \iff P^{\gamma} \,I\, \ell^{\gamma}$

  $\iff (P^{\gamma})^{\delta} \,I\, (\ell^{\gamma})^{\delta}$

  $\iff P^{\gamma\delta} \,I\, \ell^{\gamma\delta}$

We'll be looking at (P, ℓ)-central [or (P, ℓ)-axial] collineations

<u>ie</u> collineation γ that fix (don't move) P linewise and ℓ pointwise.

<u>ie</u> for every line m with $P \,I\, m$ we have $m^{\gamma} = m$ and every point Q with $Q \,I\, \ell$ we have $Q^{\gamma} = Q$

Next time: we'll aim for γ fixes some point likewise $\iff$ γ fixes some line pointwise.