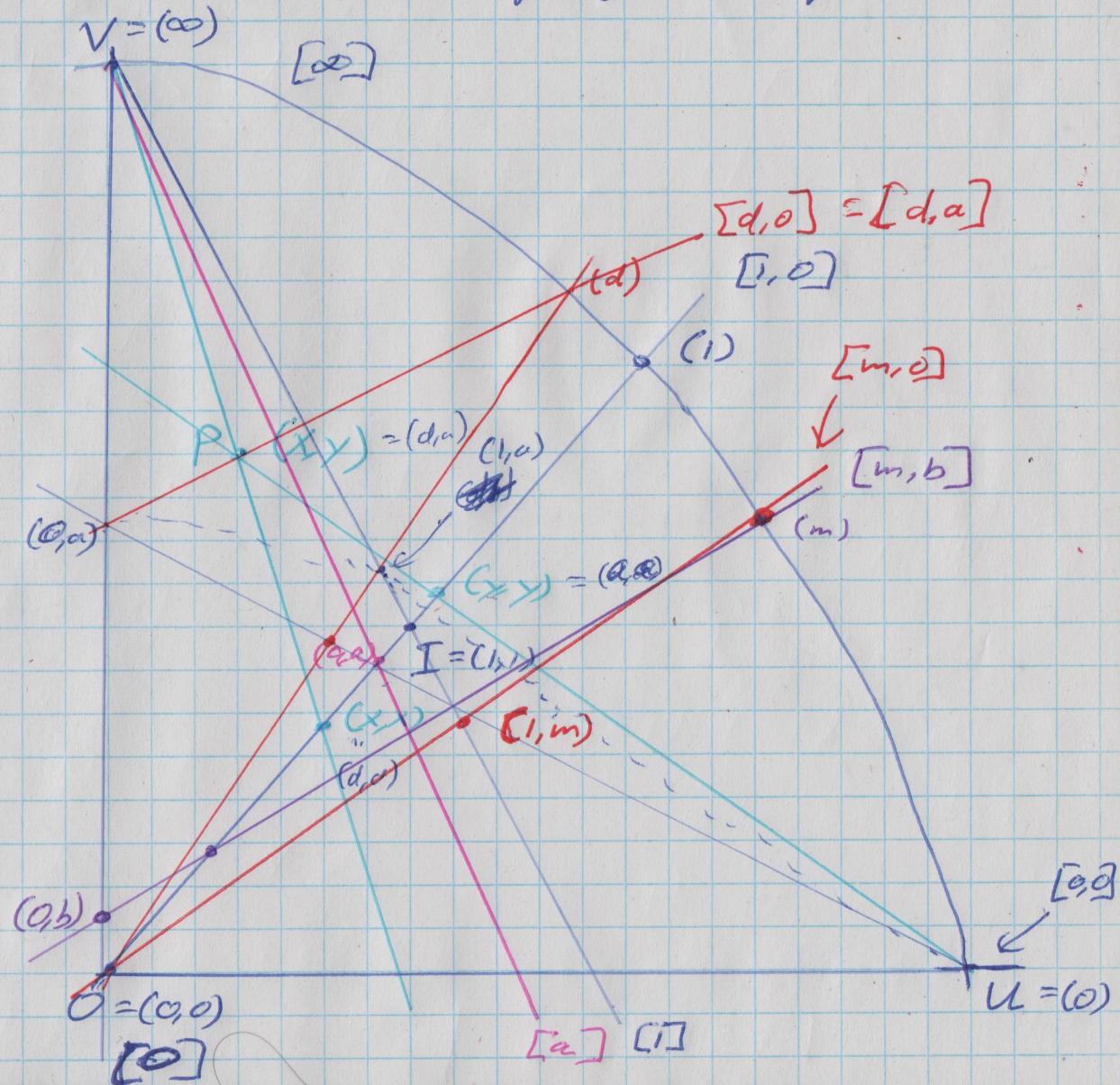


The algebraic operations

A quick recap of introducing coordinates in a projective plane:

- Pick 4 points O, I, U, V , no 3 on the same line
- Let $O = (0, 0)$, $I = (1, 1)$, $V = (\infty)$, $U = (0)$
- $OV = [0, 0]$, $OV = [0]$, $UV = [\infty]$, $OI = [1, 0]$
- Assign coordinates of the form (a, a) to all points on $[1, 0]$ other than $O, I, & (1)$.
- If P is any point not on $[\infty]$, then it gets coordinates (x, y) if $PV \cap [1, 0] = (x, x)$ and $PU \cap [1, 0] = (y, y)$.
- A line through V gets coordinate $[a]$ if it intersects OI at (a, a) .
- Other lines get coordinates $[m, b]$, if OV intersects the line at $(0, b)$ and the line $UV = [\infty]$ at (m) where $(m) \neq [\infty] \cap [m, 0]$ and $[m, 0] = (0, 0)(1, m)$.



We can now start on reverse-engineering the
actual algebraic operations. ②

We're used to saying that (x,y) is on $[m,b]$ (i.e. " $y=mx+b$ ")
if, in fact, $y=mx+b$ is true. We'd like to do
something like this - 'that actually is this' in the
case of the real projective plane or other fields.
Unfortunately not all projective planes are that nice ...

What we can do in every projective plane ~~in which~~ in which
we have introduced coordinates as above, is the following:

Define the ternary ring $T(m,x,b)$ by
 $y=T(m,x,b) \Leftrightarrow (x,y) \models [m,b]$.

Then define multiplication by

$$c \cdot d = T(c,d,0)$$

and addition by $c+d = T(1,c,d)$.

Do addition and multiplication work in familiar ways? Not necessarily, except for a few things.
 It's not even guaranteed that $T(m, x, b) = mx + b$.

1° Does multiplying by 0 give you 0?

$$a = 0 \cdot d = T(0, d, 0) \Leftrightarrow (\cancel{d}, a) \underset{\text{constant value}}{\not\in} I[0, 0] \Rightarrow$$

$$\cancel{d} = 0 \quad a = 0 \quad \text{so } 0 \cdot d = 0.$$

$$\text{and } a = d \cdot 0 = T(d, 0, 0) \Leftrightarrow (0, a) \underset{\text{constant value}}{\not\in} I[d, 0]$$

$$\Rightarrow a = 0 \quad \text{so } d \cdot 0 = 0.$$

2° Does multiplying by 1 give you the same thing back?

$$a = 1 \cdot d = T(1, d, 0) \Leftrightarrow (d, a) \underset{\text{constant value}}{\not\in} I[1, 0]$$

$$\Rightarrow d = a \quad \underline{\subseteq} \quad 1 \cdot d = d.$$

$$a = d \cdot 1 = T(d, 1, 0) \Leftrightarrow (1, a) \underset{\text{constant value}}{\not\in} I[d, 0]$$

$$\Leftrightarrow a = 1$$

Next time: We'll list the algebraic properties of $T(m, x, b)$ & take it from there.