Mathematics 3260H – Geometry II: Projective and Non-Euclidean Geometry

[Last modified 2019.07.31.] TRENT UNIVERSITY, Fall 2019 *[In Peterborough!]*

Instructor

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Department of Mathematics

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Prerequisite

MATH 1110H or 1350H. Please note that MATH 2260H – *Geometry I: Euclidean Geometry* is *not* required, though it is somewhat helpful here and there.

Text

Notes will be provided by the instructor, drawn from various sources, with links to the notes and sources on the course web page at `euclid.trentu.ca/math/sb/3260H/` .

Meetings

Lectures: Monday 14:00-14:50 in SC W1 and Thursday 13:00-14:50 in GCS 111.
Seminars: Thursday 15:00-15:50 in GCS 111.

Marking Scheme

There will be at least eleven weekly assignments and a take-home final examination. The best ten assignments will each count for 6.5% of the final mark and the final exam will count for the remaining 35%. Note that more 25% of the course marks will be obtained by the final date (Tuesday, 5 November) to withdraw from Fall half-courses without academic penalty. Assignments will not normally be accepted after the due date; students unable to hand them in on time for reasons beyond their control should contact the instructor as soon as possible.

This scheme may be modified in *exceptional* circumstances, such as a lengthy absence due to illness. Any such modification will require the agreement of both the student and the instructor.

Content & Learning Outcomes

MATH 3260H is an introduction to alternatives to Euclid's axioms, especially alternatives to the parallel postulate. Students will acquire some familiarity with basic results about hyperbolic and elliptic planes, especially the relationship between areas and interior angles of triangles, as well as the Poincaré disk and/or half-plane models of the hyperbolic plane and the antipodal sphere model of the elliptic plane. Students will also learn the axioms for affine and projective planes and basic properties of these structures, and various examples of and constructions of affine and projective planes and their properties, especially the real projective plane, as well as the introduction of coordinate systems in and transformations of projective planes, and related geometrical results, especially Desargues' and Pappus' Theorems.

Schedule

Please note that where the material covered is concerned this schedule is a polite fiction: no lesson plan survives contact with actual students unchanged! - not least because your instructor is rearranging the material this year.

Week 0. *(5–6 September)* Euclid's and Hilbert's axioms for Euclidean plane geometry. The parallel postulate and possible alternatives. No seminar this week. *Classes begin on Thursday, 5 September.*
Week 1. *(9–13 September)* Euclidean and Cartesian planes, incidence structures, affine planes, axioms for affine planes. Assignment #1 due on Thursday, 12 September.

Week 2. *(16–20 September)* The real projective plane via extended affine coordinates, via linear algebra, as well as the antipodal sphere model. Assignment #2 due on Thursday, 19 September.
Week 3. *(23–27 September)* Projective planes via extended affine coordinates and via linear algebra. Axioms for projective planes. Free completion. Assignment #3 due on Thursday, 26 September.
Week 4. *(30 September – 4 October)* Desargues' Theorem in the real projective plane and planes coordinatized by skew fields. Assignment #4 due on Thursday, 3 October.
Week 5. *(7–11 October)* Collineations, axial and central colineations. Transitivity. Assignment #5 due on Thursday, 10 October.
Week 6. *(14–18 October)* Transitivity and Desargues' Theorem. Failure of Desargues' Theorem in some projective planes. Assignment #6 due on Thursday, 17 October. *University closed on Monday, 14 October, for Thanksgiving Day.*
Fall Reading Week. *(21–25 October)* Enjoy!
Week 7. *(28 October – 1 November)* Introducing coordinates in affine planes. Ternary rings. Assignment #7 due on Thursday, 31 October.
Week 8. *(4–8 November)* Desargues' Theorem and the algebraic properties of the ternary ring. Assignment #8 due on Thursday, 7 November. *The last date to withdraw from Fall courses is Tuesday, 5 November.*
Week 9. *(11–15 November)* The metric structure of the real projective plane via the antipodal sphere model. Triangles and trigonometry in the real projective plane. Assignment #9 due on Thursday, 14 November.
Week 10. *(18–22 November.)* Elliptic geometry: the hypothesis of the obtuse angle. Incompatibility with Euclid's Postulate II. Assignment #10 due on Thursday, 21 November.
Week 11. *(25–29 November)* Hyperbolic geometry: the hypothesis of the obtuse angle. Poincaré half-plane and disk models. Compatibility with Euclid's Postulates I-IV. Assignment #11 due and take-home examination distributed on Thursday, 28 November.
Week 12. *(2–4 December)* Triangles and trigonometry in the hyperbolic plane. *Wednesday, 4 December, is the last day of classes.*
Fall Examination Period. *(6–18 December)* Take-home final examination due Thursday, 12 December.

Academic Integrity

> *Academic dishonesty, which includes plagiarism and cheating, is an extremely serious academic offence and carries penalties varying from a 0 grade on an assignment to expulsion from the University. Definitions, penalties, and procedures for dealing with plagiarism and cheating are set out in Trent University's Academic Integrity Policy. You have a responsibility to educate yourself – unfamiliarity with the policy is not an excuse. You are strongly encouraged to visit Trent's Academic Integrity website to learn more –* `www.trentu.ca/academicintegrity`

For clarity, the following guidelines will apply in MATH 3260H:

> You are permitted and encouraged to work with others and ask anyone willing (especially the instructor!) for explanations, hints, and suggestions on the assignments, and to consult whatever sources you wish. However, **all work submitted for credit must be written up entirely by yourself, giving due credit to all relevant sources of help and information. No aid may be given or received on the quizzes and final exam**, except with the intructor's permission. You may use a calculator and an aid sheet, with whatever you want on it, for the quizzes.

Access to Instruction

> *It is Trent University's intent to create an inclusive learning environment. If a student has a disability and/or health consideration and feels that he/she may need accommodations to succeed in this course, the student should contact the Student Accessibility Services Office (SAS), Blackburn Hall Suite 132, 705 748-1281,* `sas@trentu.ca`*. For Trent University in Oshawa Student Accessibility Services Office contact 905 435-5102, ext. 5024. Complete text can be found under Access to Instruction in the Academic Calendar.*

Web Page

This course will make only minimal use of Blackboard/LearningSystem. In particular, marks will not be posted to Blackboard – please pick up your marked work in class or office hours so as to learn from your mistakes. It will, however, have a web page with hopefully up-to-date information and links to all the assignments and other handouts at `euclid.trentu.ca/math/sb/3260H/` . This page also has links to material from several past iterations of the course.