Mathematics 3260H – Geometry II: Projective and Non-Euclidean Geometry
Trent University, Fall 2019

Assignment #9
Linear algebra?!
Due on Monday, 11 November.

Recall that we showed in class how to introduce extended affine coordinates into an arbitrary projective plane and define an associated *ternary ring* as follows:

> Suppose that $\pi = (\mathcal{P}, \mathcal{L}, \mathbf{I})$ is a projective plane and that R is a set of symbols, including 0 and 1, which is just large enough to assign a symbol from R to each point of some line in the affine plane corresponding to π. (That is, $|\mathcal{P}| = |R| + 1$.)
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> Choose a quadrangle $OEUV$ in π (the *fundamental quadrangle* of the coordinate system) and declare their coordinates to be $O = (0, 0)$, $E = (1, 1)$, $U = (0)$, and $V = (\infty)$. Give every other point on the line OE coordinates of the form (a, a) for some distinct a in $R \setminus \{0, 1\}$. For any point X not incident with OE or UV, we assign coordinates by setting $X = (a, b)$ if $XV \cap OE = (a, a)$ and $XU \cap OE = (b, b)$. Finally, give each point Y on UV, other than U or V, coordinates by setting $Y = (m)$ if $OY \cap EV = (1, m)$.
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> For each m, $k \in R$, the line joining the point $(0, k)$ and the point (m) is given coordinates $[m, k]$, and for each $a \in R$, the line joining $V = (\infty)$ to (a, a) is given the coordinate $[a]$. Finally, UV is given the coordinate $[\infty]$.
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> The ternary operation T of the corresponding ternary ring is defined by setting $y = T(m, x, k)$ if and only if $(x, y) \, \mathbf{I} \, [m, k]$. We can then define $+$ and $\cdot$ on R by $a + b = T(1, a, b)$ and $a \cdot b = T(a, b, 0)$. A ternary ring is *linear* if we have $T(m, x, k) = (m \cdot x) + k$ for all m, x, $k \in R$.

1. Suppose a projective plane with extended affine coordinates is $(U, OV) = ((0), [0])$-transitive. Show that the corresponding ternary ring is linear. *[10]*