Mathematics 3260H – Geometry II: Projective and Non-Euclidean Geometry
TRENT UNIVERSITY, Fall 2019

Assignment #5
Desargues' Theorem in the Real Projective Plane
Due on Thursday, 3 October.

Recall the following from Assignment #4:

DEFINITION. Two triangles ABC and DEF are said to be *in perspective from a point* P if AD, BE, and CF are all incident with P, and *in perspective from a line* ℓ if $AB \cap DE$, $BC \cap EF$, and $AE \cap DF$ are all incident with ℓ.

We can use this definition to conveniently state the following result:

> DESARGUES' THEOREM. Two triangles are in perspective from some point if and only if they are in perspective from some line.

1. Sketch the diagrams for Desargues' Theorem for the cases where the perspective point is on the perspective line and where the perspective point is not on the perspective line. *[3]*

2. Prove that Desargues' Theorem holds in the real projective plane. *[7]*

Hint: If you use the presentation of the real projective plane via projective coordinates and prove either direction of the "if and only if", you get the other direction for free. On the other hand, no matter which presentation you pick, you'll probably have to work to get either direction ...

NOTE: Desargues' Theorem is not always true in every projective plane. It turns out to be intimately related to the existence of central collineations, and a projective plane without enough central collineations will have Desargues' Theorem fail at least some of the time. Having Desargues' Theorem be always true in a projective plane turns out to be equivalent to the plane being coordinatized by a skew field or a field.