

Mathematics 3260H – Geometry II: Projective and Non-Euclidean Geometry
TRENT UNIVERSITY, Fall 2019

Assignment #2
The Real Projective Plane via Linear Algebra
Due on Thursday, 19 September.

Recall from class that a projective plane is a geometry consisting of a set of points and a set of lines satisfying the following axioms:

- PI.** Any two distinct points are connected by a unique line.
- PII.** Any two distinct lines intersect in a unique point.
- PIII.** There exist four points such that no three are on the same line.

The following is one of several equivalent methods of defining the real projective plane.

- The points of the plane are the one-dimensional subspaces of \mathbb{R}^3 .
- The lines of the plane are the two-dimensional subspaces of \mathbb{R}^3 .
- A point of the plane is incident with a line of the plane exactly when the one-dimensional subspace is contained in the two-dimensional subspace.

1. Verify that the real projective plane so defined is indeed a projective plane. [10]

Hint. You should probably do this by directly showing that this construction satisfies the three axioms for a projective plane. If you really must do it by showing that this way of defining the real projective plane is equivalent to constructing the real projective plane using extended affine coordinates, that is probably best done indirectly, such as by showing that each of the two methods is equivalent to a suitable third method ...