

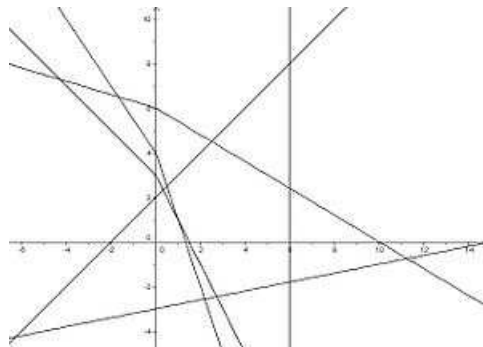
Mathematics 3260H – Geometry II: Projective and Non-Euclidean Geometry
TRENT UNIVERSITY, Fall 2019

Assignment #1
A modified Cartesian plane
Due on Thursday, 12 September.

An affine plane is a geometry consisting of a set of points and a set of lines satisfying the following axioms:

- AI.** Any two distinct points are connected by a unique line.
- AII.** Given a line ℓ and a point P not on ℓ , there is a unique line m through P that has no points in common with ℓ .
- AIII.** There exist three points that are not all on the same line.

The *Moulton plane* is the affine plane obtained from the Cartesian plane by replacing straight lines with negative slope by lines which bend to double the slope as they cross the y -axis from left to right.



More formally:

- The points of the Moulton plane are the points of the Cartesian plane \mathbb{R}^2 .
- The lines of the Moulton plane include:
 - The vertical lines of the Cartesian plane, *i.e.* $x = c$ for each $c \in \mathbb{R}$.
 - The lines of non-negative slope of the Cartesian plane, *i.e.* $y = mx + b$ for $m, b \in \mathbb{R}$ with $m \geq 0$.
 - The bent lines given by $y = \begin{cases} mx + b & x \leq 0 \\ 2mx + b & x \geq 0 \end{cases}$ for $m, b \in \mathbb{R}$ with $m \leq 0$.
- A point is on a line of the Moulton plane exactly when its Cartesian coordinates satisfy the equation of the line.

1. Verify that the Moulton plane is indeed an affine plane. [10]