ELEMENTS BOOK 1

Definitions

1 A point is that of which there is no part.

2 And a line is a length without breadth.

3 And the extremities of a line are points.

4 A straight-line is whatever lies evenly with points upon itself.

5 And a surface is that which has length and breadth alone.

6 And the extremities of a surface are lines.

7 A plane surface is whatever lies evenly with straight-lines upon itself.

8 And a plane angle is the inclination of the lines, when two lines in a plane meet one another, and are not laid down straight-on with respect to one another.

9 And when the lines containing the angle are straight then the angle is called rectilinear.

10 And when a straight-line stood upon (another) straight-line makes adjacent angles (which are) equal to one another, each of the equal angles is a right-angle, and the former straight-line is called perpendicular to that upon which it stands.

11 An obtuse angle is greater than a right-angle.

12 And an acute angle is less than a right-angle.

13 A boundary is that which is the extremity of something.

14 A figure is that which is contained by some boundary or boundaries.

15 A circle is a plane figure contained by a single line [which is called a circumference], (such that) all of the straight-lines radiating towards [the circumference] from a single point lying inside the figure are equal to one another.

16 And the point is called the center of the circle.

17 And a diameter of the circle is any straight-line, being drawn through the center, which is brought to an end in each direction by the circumference of the circle. And any such (straight-line) cuts the circle in half.[1]

18 And a semi-circle is the figure contained by the diameter and the circumference it cuts off. And the center of the semi-circle is the same (point) as (the center of) the circle.

19 Rectilinear figures are those figures contained by straight-lines: trilateral figures being contained by three straight-lines, quadrilateral by four, and multilateral by more than four.

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[1]This should really be counted as a postulate, rather than as part of a definition.

20 And of the trilateral figures: an equilateral triangle is that having three equal sides, an isosceles (triangle) that having only two equal sides, and a scalene (triangle) that having three unequal sides.

21 And further of the trilateral figures: a right-angled triangle is that having a right-angle, an obtuse-angled (triangle) that having an obtuse angle, and an acute-angled (triangle) that having three acute angles.

22 And of the quadrilateral figures: a square is that which is right-angled and equilateral, a rectangle that which is right-angled but not equilateral, a rhombus that which is equilateral but not right-angled, and a rhomboid that having opposite sides and angles equal to one another which is neither right-angled nor equilateral. And let quadrilateral figures besides these be called trapezia.

23 Parallel lines are straight-lines which, being in the same plane, and being produced to infinity in each direction, meet with one another in neither (of these directions).

Postulates

1 Let it have been postulated to draw a straight-line from any point to any point.

2 And to produce a finite straight-line continuously in a straight-line.

3 And to draw a circle with any center and radius.

4 And that all right-angles are equal to one another.

5 And that if a straight-line falling across two (other) straight-lines makes internal angles on the same side (of itself) less than two right-angles, being produced to infinity, the two (other) straight-lines meet on that side (of the original straight-line) that the (internal angles) are less than two right-angles (and do not meet on the other side).[2]

Common Notions

1 Things equal to the same thing are also equal to one another.

2 And if equal things are added to equal things then the wholes are equal.

3 And if equal things are subtracted from equal things then the remainders are equal.[3]

4 And things coinciding with one another are equal to one another.
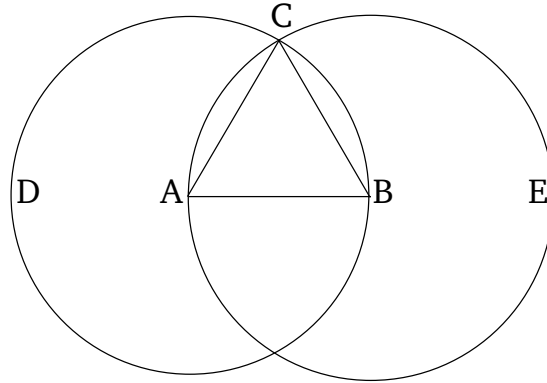
5 And the whole [is] greater than the part.

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[2]This postulate effectively specifies that we are dealing with the geometry of *flat*, rather than curved, space.

[3]As an obvious extension of C.N.s 2 & 3—if equal things are added or subtracted from the two sides of an inequality then the inequality remains an inequality of the same type.

Proposition 1

To construct an equilateral triangle on a given finite straight-line.

Let AB be the given finite straight-line.

So it is required to construct an equilateral triangle on the straight-line AB.

Let the circle BCD with center A and radius AB have been drawn [Post. 3], and again let the circle ACE with center B and radius BA have been drawn [Post. 3]. And let the straight-lines CA and CB have been joined from the point C, where the circles cut one another,[4] to the points A and B (respectively) [Post. 1].

And since the point A is the center of the circle CDB, AC is equal to AB [Def. 1.15]. Again, since the point B is the center of the circle CAE, BC is equal to BA [Def. 1.15]. But CA was also shown (to be) equal to AB. Thus, CA and CB are each equal to AB. But things equal to the same thing are also equal to one another [C.N. 1]. Thus, CA is also equal to CB. Thus, the three (straight-lines) CA, AB, and BC are equal to one another.

Thus, the triangle ABC is equilateral, and has been constructed on the given finite straight-line AB. (Which is) the very thing it was required to do.

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[4]The assumption that the circles do indeed cut one another should be counted as an additional postulate. There is also an implicit assumption that two straight-lines cannot share a common segment.