Mathematics 3260H – Geometry II: Projective and non-Euclidean geometry
TRENT UNIVERSITY, Fall 2017

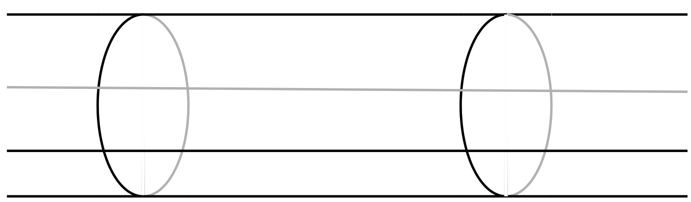
Quizzes

Quiz #1. Wednesday, 20 September, 2017. *[15 minutes]*

Consider the following model of a geometry. The points of the model are the points (of the surface) of an infinite cylinder with a circular cross-section (living in three-dimensional Euclidean space). The lines of the model come in two types:

i. Euclidean straight lines parallel to the axis of the cylinder on (the surface of) the cylinder, and
ii. Euclidean circles perpendicular to the axis of the cylinder on (the surface of) of the cylinder.

Angles and lengths of curves are measured as they would be in three-dimensional Euclidean space.



1. Explain which of Euclid's Postulates I–IV is true in this model of a geometry. *[4]*
2. Is Playfair's Axiom true in this model of a geometry? Why or why not? *[1]*

Quiz #2. Wednesday, 27 September, 2017. *[15 minutes]*

1. In the Euclidean plane the collection of points which are one side of and a constant distance from a given line also form a line. Explain why this doesn't work in one of the Poincaré half-plane model of the hyperbolic plane or the antipodal sphere model of the elliptic plane. (Just one!) *[5]*

Take-home Quiz #3. Wednesday, 4 October, 2017. *[1 day]*

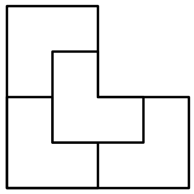
1. Devise your own quiz question about non-Euclidean geometry. It should be doable in 10–20 minutes (you get to indicate the time limit), and it would be a good idea to indicate how a suitable solution would work. *[5]*

Quiz #4. Wednesday, 11 October, 2017. *[20 minutes]*

1. Suppose $\triangle ABC$ in the elliptic plane is equilateral and has a right angle for each one of its three internal angles. Can $\triangle ABC$ be subdivided into smaller, but still equilateral, triangles? Show how it can or show that it cannot be so subdivided. *[5]*

Take-home Quiz #5. Wednesday, 18 October, 2017. *[1 day]*

A two-dimensional shape, such as a polygon, is a *reptile* if it can be partitioned into a finite number of smaller copies of itself. For example, the L-shaped tromino obtained by sticking three squares together edge to edge, but not in a straight line, is a reptile:



1. Explain why no regular polygons are reptiles in the elliptic plane or in the hyperbolic plane. *[2.5]*
2. Which regular polygons are reptiles in the Euclidean plane? Explain why. *[2.5]*

Quiz #6. Wednesday, 1 November, 2017. *[12 minutes]*

1. Suppose a "geometry" satisfies the following axioms:
   i. Every pair of points is connected by exactly two lines.
   ii. Every pair of lines intersect in exactly two points.
   iii. It is not the case that there are three points which are not all on one line.

   Find all such "geometries" that you can. Draw the pictures! *[5]*

Quiz #7. Wednesday, 8 November, 2017. *[12 minutes]*

Do *one* (1) of the following two problems.

1. Consider the triangle in the real projective plane whose vertices have projective coordinates $(1, 1, 1)$, $(1, 2, 1)$, and $(2, 1, 1)$. Find the projective coordinates of the lines forming the sides of the triangle. *[5]*
2. Suppose that a point and all the lines through it (but not the other points on those lines) are removed from a projective plane. Which of axioms **I–III** of a projective plane does the resulting structure still have to satisfy? Why? *[5]*

Quiz #8. Wednesday, 15 November, 2017. *[15 minutes]*

1. Suppose β is a collineation of a projective plane with centre P and axis ℓ, and suppose X and Y are points not on ℓ and different from P such that $X^\beta = Y$. If Z is any point of the plane not on either of the lines ℓ and XY, explain how to locate Z^β. *[5]*

Quiz #9. Wednesday, 22 November, 2017. *[10 minutes]*

1. Suppose γ is a collineation of a projective plane which is its own inverse, *i.e.* $\gamma^2 = \gamma \circ \gamma$ is the identity map. Show that γ must fix at least one point. *[5]*

Take-home Quiz #10. Wednesday, 29 November, 2017. *[1 day]*

1. Suppose δ is a collineation of a projective plane that fixes all four vertices of quadrangle $ABCD$, but is not the identity. Show that the collection of all points and lines of the plane that are fixed by δ, with incidence between them as in the plane, is itself a projective plane.

Quiz #11. Wednesday, 6 December, 2017. *[20 minutes]*

1. Why isn't the hyperbolic plane a projective plane? *[2.5]*
2. Why doesn't the method for extending the Euclidean plane to a projective plane by adding points and a line "at infinity" work for the hyperbolic plane? *[2.5]*