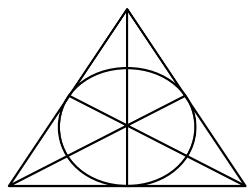
Mathematics 3260H – Geometry II: Projective and non-Euclidean geometry
Trent University, Fall 2017

Assignment #8
Some Collineations
Due on Wednesday, 15 November.

Recall that the Fano configuration is the smallest projective plane, consisting of seven points and seven lines, with each point incident with three lines and each line incident with three points.



1. Suppose ℓ is a line of the Fano configuration, P a point not incident with ℓ, and α is a collineation of the Fano configuration which fixes every point on ℓ and every line through ℓ. Show that α is the identity collineation. *[5]*

For the next problem, you will need to recall how to use projective coordinates in the real projective plane.

2. Suppose $P = (u, v, w)$ and $Q = (x, y, z)$ are points of the real projective plane which are *not* on the lines $\ell = [a, b, c]$ and $m = [d, e, f]$, respectively. Show that there is a collineation δ of the real projective plane such that $P^\delta = Q$ and $\ell^\delta = m$. *[7]*

Note: It's perfectly possible to have P be on m and/or Q be on ℓ here.

Hint: A suitable linear transformation of $\mathbb{R}^3$ will give such a collineation ...