Mathematics 3260H – Geometry II: Projective and non-Euclidean geometry

Assignment #7
Affine vs. Projective Planes
Due on Wednesday, 8 November.

Recall from class that an affine plane is a geometry consisting of a set of points and lines, and a relation of incidence between them, satisfying the following axioms:

AI. Any two distinct points are connected by an unique line.

AII. Given a line ℓ and a point P not on ℓ, there is an unique line m through P that has no points in common with ℓ.

AIII. There exist three points that are not all on the same line.

Recall also that a projective plane is a geometry consisting of a set of points and lines, and a relation of incidence between them, satisfying the following axioms:

I. Any two distinct points are connected by an unique line.

II. Any two distinct lines meet at an uniqie point.

III. There exist four points such that that no three are on the same line.

It turns out that these definitions are very closely related, as one can convert one type of plane into the other by a fairly simple procedure.

1. Suppose you are given a projective plane and proceed to remove a single line and all the points on this line from the given plane. Show that what is left over is an affine plane. *[5]*

2. Suppose you are given an affine plane. Explain how to turn it into a projective plane by adding a new line with new points on it. *[5]*

   Hint: The key is how the new points interact with the old lines.