

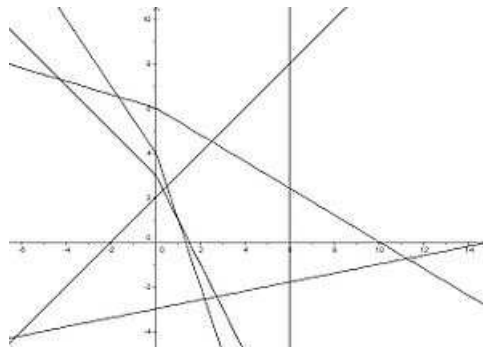
Mathematics 3260H – Geometry II: Projective and non-Euclidean geometry  
TRENT UNIVERSITY, Fall 2017

Assignment #6  
A modified Cartesian plane  
Due on Wednesday, 1 November.

Recall from class that an affine plane is a geometry consisting of a set of points and lines satisfying the following axioms:

- AI.** Any two distinct points are connected by a unique line.
- AII.** Given a line  $\ell$  and a point  $P$  not on  $\ell$ , there is a unique line  $m$  through  $P$  that has no points in common with  $\ell$ .
- AIII.** There exist three points that are not all on the same line.

The *Moulton plane* is the affine plane obtained from the Cartesian plane by replacing straight lines with negative slope by lines which bend to double the slope as they cross the  $y$ -axis from left to right.



More formally:

- The points of the Moulton plane are the points of the Cartesian plane  $\mathbb{R}^2$ .
- The lines of the Moulton plane include:
  - The vertical lines of the Cartesian plane, *i.e.*  $x = c$  for each  $c \in \mathbb{R}$ .
  - The lines of non-negative slope of the Cartesian plane, *i.e.*  $y = mx + b$  for  $m, b \in \mathbb{R}$  with  $m \geq 0$ .
  - The bent lines given by  $y = \begin{cases} mx + b & x \leq 0 \\ 2mx + b & x \geq 0 \end{cases}$  for  $m, b \in \mathbb{R}$  with  $m \leq 0$ .
- A point is on a line of the Moulton plane exactly when its Cartesian coordinates satisfy the equation of the line.

1. Verify that the Moulton plane is indeed an affine plane. [10]