Mathematics 3260H – Geometry II: Projective and non-Euclidean geometry
TRENT UNIVERSITY, Fall 2017

Assignment #4
Congruence in the hyperbolic plane
Due on Wednesday, 11 October.

By way of notation, let the sides opposite the vertices A, B, and C of $\triangle ABC$ have lengths a, b, and c, respectively, and denote the interior angles at the vertices by α, β, and γ respectively. Recall from class that if $\triangle ABC$ is a triangle of the hyperbolic plane, then:

$$\begin{aligned}
\cosh(a) &= \cosh(b)\cosh(c) - \cos(\alpha)\sinh(b)\sinh(c) \\
\cos(\alpha) &= \cos(a)\sin(\beta)\sin(\gamma) - \cos(\beta)\cos(\gamma) \\
\frac{\sin(\alpha)}{\sinh(a)} &= \frac{\sin(\beta)}{\sinh(b)} = \frac{\sin(\gamma)}{\sinh(c)}
\end{aligned}$$

Similar equations, *mutatis mutandis*, hold if we interchange the roles of the various angles and the sides opposite to them. Moreover, the area of $\triangle ABC$ is $\pi - \alpha - \beta - \gamma$ (measuring angles in radians and with a suitable choice of units for length).

1. Does each of the following congruence criteria work in the hyperbolic plane? Justify your answer as best you can in each case. *[10 = 5 × 2 each]*

i. SSS *ii.* SAS *iii.* ASA *iv.* SAA *v.* AAA [S = Side and A = Angle]