

Mathematics 3260H – Geometry II: Projective and non-Euclidean geometry
TRENT UNIVERSITY, Fall 2017

Assignment #1
With apologies to Prof. Tolkien ...
Due on Wednesday, 20 September.

If the Númenoreans had been mathematicians, perhaps the rhyme of lore* Gandalf quotes to Pippin during the ride from Rohan to Gondor in the *The Lord of the Rings* would have been something like:

*Tall ships and tall kings
Three times three,
What brought they from the foundered land
Over the flowing sea?
Seven points and seven lines
In one geometry:
Every point met three lines,
Every line met points three,
Every pair of points connected,
Every line pair intersected.*

The creator of the seven stones of the original verse is supposed to have been the elf Fëanor, who had two half-brothers named Fingolfin and Finarfin. Credit for this geometry in this alternate universe would go to his previously unknown sibling Fano ...

1. Draw a picture of this alternate universe Númenorean geometry. [3]
2. Can this geometry be drawn in the Euclidean plane with all of its “lines” being straight line segments of the Euclidean plane? Give an example or prove it can’t be done. [4]
3. Is there a *quadrilateral* - four lines, no three of which pass through the same point - in this geometry? Give an example or show that there isn’t one. [1]
4. Consider the following adaptation of a more famous verse[†] from the *Lord of the Rings*.
One line to meet them all, at infinity to find them,
One line to join them all and at some points to bind them,
In projective planes where no parallels lie.

What mathematical object(s) and/or operation(s) does this verse allude to? [2]

Bonus: What does this geometry have to do with the world of J.K. Rowling’s *Harry Potter*? [0.5]

* “Tall ships and tall kings/ Three times three,/ What brought they from the foundered land/ Over the flowing sea?/ Seven stars and seven stones/ And one white tree.”

† “One Ring to rule them all, One Ring to find them, / One Ring to bring them all and in the darkness bind them, / In the Land of Mordor where the Shadows lie.”